What is experimental mathematics?

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Jonathan M. Borwein, 1951–2016

- 388 published journal articles; another 103 in refereed conference proceedings.
- ISI Web of Knowledge lists 6,593 citations from 351 items; one paper has been cited 666 times.
- His work spanned pure mathematics, applied mathematics, optimization theory, computer science, mathematical finance, and experimental mathematics.
- Borwein sought to do research that is accessible, and to highlight aspects of his work that a broad audience (including both researchers and the lay public) could appreciate.
- More information, including memorials and links to nearly 1700 publications, preprints and talks: http://www.jonborwein.org.



What is experimental mathematics?

Experimental mathematics encompasses using a computer to:

- Gain insight and intuition about mathematical phenomena.
- Explore specific mathematical questions.
- > Plot or otherwise visually represent mathematical objects.
- Test (and falsify) conjectured hypotheses.
- Discover new mathematical identities and theorems.
- > Find rigorous proofs of empirically discovered hypotheses.

In one sense, experimental mathematics is nothing new — all of the famous historical mathematicians (Newton, Euler, Gauss, Ramanujan, many others) employed experimentation and computational discovery in their work.

The advent of the computer has fundamentally and permanently changed the methodology of doing mathematics.

But the overall goal remains the same: to find and rigorously establish secure mathematical knowledge.

Von Neumann's warning about the future of mathematics

Experimental mathematics provides a means to escape the trap feared by John von Neumann when he wrote,

But there is a grave danger that the subject [of mathematics] will develop along the line of least resistance, that the stream, so far from its source, will separate into a multitude of insignificant branches, and that the discipline will become a disorganized mass of details and complexities. In other words, at a great distance from its empirical source, or after much "abstract" inbreeding, a mathematical subject is in danger of degeneration. . . .

In any event, whenever this stage is reached, the only remedy seems to me to be the rejuvenating return to the source: the re-injection of more or less directly empirical ideas. I am convinced that this was a necessary condition to conserve the freshness and the vitality of the subject and that this will remain equally true in the future.

Tools of experimental mathematics

Experimental mathematicians use these and other tools:

- Symbolic math software, such as Mathematica, Maple and Sage.
- Online facilities such as:
 - The Online Encyclopedia of Integer Sequences: http://www.oeis.org.
 - The Inverse Symbolic Calculator 2: https://isc.carma.newcastle.edu.au.
- ► Custom-written software, typically in C++, Python or Fortran.
- High-precision arithmetic software packages, provided in *Mathematica*, *Maple* and *Sage*, but also in packages such as MPFR and GMP.
- ► Software to facilitate highly parallel computations, such as OpenMP and MPI.
- ► Graphics and visualization software, provided in *Mathematica*, *Maple* and *Sage*, but also in packages such as *Cinderella*.
- Specialized math software for particular fields, such as numerical analysis, algebra, topology.

Experimental mathematics and outreach

Experimental mathematics provides a unique opportunity to involve a much broader community in doing real mathematical research:

- High school and college students.
- Computer scientists.
- Computer graphics experts.
- Statisticians.
- Data scientists.

We should find ways to involve computational methods in every aspect of mathematical teaching and research.

$\boldsymbol{\pi}$ and experimental mathematics

 π continues to excite millions worldwide, leading many to pursue careers in math, science and engineering.



A random walk on the first 100 billion base-4 digits of π



This dataset can be explored online: http://gigapan.com/gigapans/106803

Winning the battle, but losing the war

Mathematicians and scientists may be winning battles to publish papers and obtain grants, but we are losing the war for the hearts and minds of the public:

- ▶ 51% in U.S. (54% in Australia) either do not believe in climate change, or do not believe there is any human connection.
- ▶ 42% in U.S. (23% in Australia) believe that humans were created within past 10,000 years.
- ▶ 38% in U.S. (32% in Australia) do not believe in evolution.
- ▶ 32% in U.S. (24% in Australia) do not believe vaccinations are safe.
- ▶ 48% in U.S. (34% in Australia) believe humans are being visited by extraterrestrial UFOs.
- ▶ 6% in U.S. believe NASA faked the Apollo moon landings.
- Some even dispute the value of π . (I frequently receive such email.)

Anti-science movements arise from both sides of the political spectrum:

- ▶ From the left: anti-vaccination and anti-fluoridation.
- From the right: anti-climate change and anti-evolution.

Carl Sagan's warning (The Demon Haunted World, 1995)

I have a foreboding of an America in my children's or my grandchildren's time — when the United States is a service and information economy; when nearly all the key manufacturing industries have slipped away to other countries; when awesome technological powers are in the hands of a very few, and no one representing the public interest can even grasp the issues; when the people have lost the ability to set their own agendas or knowledgeably question those in authority; when, clutching our crystals and nervously consulting our horoscopes, our critical faculties in decline, unable to distinguish between what feels good and what's true, we slide, almost without noticing, back into superstition and darkness. ...

We've arranged a global civilization in which most crucial elements ... profoundly depend on science and technology. We have also arranged things so that almost no one understands science and technology. This is a prescription for disaster. We might get away with it for a while, but sooner or later this combustible mixture of ignorance and power is going to blow up in our faces.

How can we turn the tide?

- Start a blog.
- Visit schools or give lectures.
- Write books for the general public.
- Write articles for science news forums.
- Write expository articles for scientific journals.
- Pursue research topics that have potentially wide appeal.
- Recognize communication skills in hiring, promotion and research grant decisions.
- Find ways to utilize computers and otherwise make teaching and research much more engaging and interesting.



2009, the U.S. House of Representatives passed resektion efficiely designating March 13 as an "National IP Day," and encouraging "schools and educators to observe th my with appropriate activities that its task's maderia along H and engage them along the study of mathematics," This may well be the First legislation on H Day to have been been day and inclus developmentarial Day.

n general, at is much more in the public eye than it was even five or ten years ago. On May 9, 2013, the North American gaiz shew Jospandy! featured an entire category e puestions on pt. The clues provided were:

- 1. (\$200) PI is the ratio of this measurement of a circle to its diameter.
- (\$400) Numerically, pl is considered this, like a type of "meditation".
 (\$500) Per about \$10,100 x pl, this "Black Swan" director made "PV", his 1998 debut film about a meth whip
- (\$900) In the 100s A.D. Uts Absandran astronomer calculated a more process value of pr, the equivalent of 3.1.

(block) if the cost for the area of this avail geometric shape with prix A x B, if A B are half of its longest B shortest dame

te clues and the answers (all were answered connectly by various contestants) are given here in the 3 archive, an independent repository is uns and answers maintained by Jeopandyt films.

Some other recent examples of the public's mania for pl include the follow

- On September 12, 2012, five arrowth armed with dot-matrix-state signeriting technology wrote 1000 digits of pl in the sky above the San Francisco Birk Area as a saectacular if castly since of pfloritance art.
- In President pay Prevent in Indextancian in usary previous previous previous previous and the previous of t
- On August 18, 2005, Google effored 14,159,265 "new sloss of rich technology" in their initial public stock effering. On January 29, 2013 they offered a pi relition dati prize for successful hacking of the Ownere Operating System on a specific Android phone.