Computer discovery and analysis of large Poisson polynomials

David H. Bailey
http://www.davidhbailey.com
Lawrence Berkeley National Lab (retired)
University of California, Davis, Dept. of Computer Science

Collaborators:
Jonathan M. Borwein (deceased 2 Aug 2016), University of Newcastle, Australia
Jason Kimberley, University of Newcastle, Australia
Watson Ladd, University of California, Berkeley

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Jonathan M. Borwein, 1951–2016

- 388 published journal articles; another 103 in refereed conference proceedings.
- ISI Web of Knowledge lists 6,593 citations from 351 items; one paper has been cited 666 times.
- His work spanned pure mathematics, applied mathematics, optimization theory, computer science, mathematical finance, and experimental mathematics.
- Borwein sought to do research that is accessible, and to highlight aspects of his work that a broad audience (including both researchers and the lay public) could appreciate.
- More information, including memorials and links to nearly 1700 publications, preprints and talks: http://www.jonborwein.org.

Standing on the shoulders of giants

The following study is a paradigm of multidisciplinary experimental mathematics, as it crucially relies on many highly talented contributions:

- Work by Brent, Zimmermann, Lefevre and other developers of the MPFR package, which was used for extreme precision computation.
- An enormous software infrastructure behind our computer code:
  - GNU compilers and Apple's Berkeley Unix software.
  - Fortran custom datatypes and operator overloading.
  - OpenMP software for parallel processing.
- Ferguson's PSLQ algorithm (as far as we are aware, this study involves the largest computations ever done using PSLQ).
- Crandall's work applying the Poisson equation to image enhancement.
- Jon Bowein's derivation of a much more rapidly convergent algorithm for the Poisson phi function.
- Numerous studies involving elliptic curves, theta functions, ideals and fields.
- A key observation by Jason Kimberley of the University of Newcastle, Australia.
- A concluding proof by Watson Ladd, a graduate student at U.C. Berkeley.

The PSLQ integer relation algorithm

Given a vector (x_n) of real numbers, an integer relation algorithm finds integers (a_n) such that

$$a_1 x_1 + a_2 x_2 + \cdots + a_n x_n = 0$$

(to within the precision of the arithmetic being used), or else finds bounds within which no relation can exist.

Helaman Ferguson's PSLQ algorithm is the most widely used integer relation algorithm, although variants of the LLL algorithm can also be used.

Integer relation detection (using PSLQ or any other algorithm) requires very high numeric precision, both in the input data and in the operation of the algorithm.

1. H. R. P. Ferguson, D. H. Bailey and S. Arno, "Analysis of PSLQ, an integer relation finding algorithm," *Mathematics of Computation*, vol. 68, no. 225 (Jan 1999), 351–369.
2. D. H. Bailey and D. J. Broadhurst, "Parallel integer relation detection: Techniques and applications," *Mathematics of Computation*, vol. 70, no. 236 (Oct 2000), 1719–1736.

Helaman Ferguson's "Umbilic Torus SC" sculpture at Stony Brook Univ.

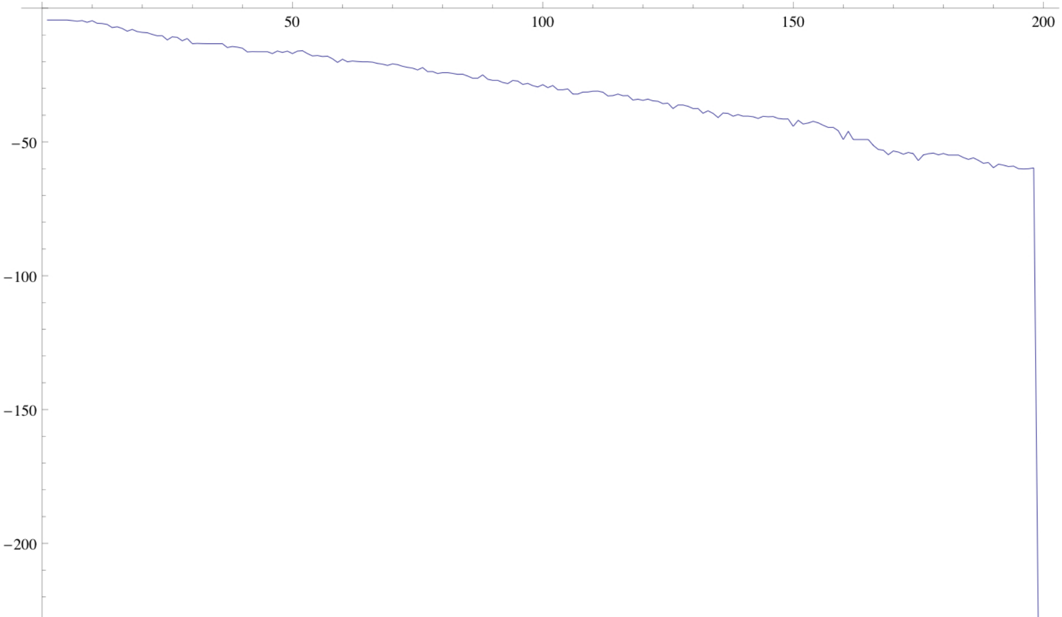
PSLQ, continued

- PSLQ constructs a sequence of integer-valued matrices B_n that reduce the vector $y = x \cdot B_n$, until either the relation is found (as one of the columns of matrix B_n), or else precision is exhausted.
- A relation is detected when the size of smallest entry of the y vector suddenly drops to roughly "epsilon" (i.e. 10^{-p}, where p precision in digits).
- The size of this drop can be viewed as a "confidence level" that the relation is not a numerical artifact: a drop of 20+ orders of magnitude almost always indicates a real relation.

Efficient variants of PSLQ:

- 2-level and 3-level PSLQ perform almost all iterations with only double precision, updating full-precision arrays as needed. They are hundreds of times faster than the original PSLQ.
- Multi-pair PSLQ dramatically reduces the number of iterations required. It was designed for parallel systems, but runs faster even on 1 CPU.

Decrease of $\log 10(\min |y_i|)$ in multipair PSLQ run



Application of multipair PSLQ

One simple but important application of multipair PSLQ is to recognize a computed numerical value as the root of an integer polynomial of degree m.

Example: The following constant is suspected to be an algebraic number:

$\alpha = 1.232688913061443445331472869611255647068988824547930576057634684778\ldots$

What is its minimal polynomial?

Method: Compute the vector $(1, \alpha, \alpha^2, \cdots, \alpha^m)$ for $m = 30$, then input this vector to multipair PSLQ.

Answer (using 250-digit arithmetic):

$$\begin{aligned} 0 = {} & 697 - 1440\alpha - 20520\alpha^2 - 98280\alpha^3 - 102060\alpha^4 - 1458\alpha^5 + 80\alpha^6 - 43920\alpha^7 \\ & + 538380\alpha^8 - 336420\alpha^9 + 1215\alpha^{10} - 80\alpha^{12} - 56160\alpha^{13} - 135540\alpha^{14} - 540\alpha^{15} \\ & + 40\alpha^{18} - 7380\alpha^{19} + 135\alpha^{20} - 10\alpha^{24} - 18\alpha^{25} + \alpha^{30} \end{aligned}$$

The Poisson potential function

In 2012, Richard Crandall, while investigating techniques to sharpen images, noted that each pixel was given by a form of the 2-D Poisson potential function:

$$\phi_2(x, y) = \frac{1}{\pi^2} \sum_{m,n \text{ odd}} \frac{\cos(m\pi x)\cos(n\pi y)}{m^2 + n^2}$$

In a 2013 study, we numerically discovered, and then proved the intriguing fact that for rational (x, y),

$$\phi_2(x, y) = \frac{1}{\pi} \log \alpha$$

where α is *algebraic*, i.e., the root of a some integer polynomial of degree m.

By computing high-precision numerical values of $\phi_2(x, y)$ for various specific rational x and y, and applying a multipair PSLQ program, we were able to produce the explicit minimal polynomials for α in numerous specific cases.

- D. H. Bailey, J. M. Borwein, R. E. Crandall and J. Zucker, "Lattice sums arising from the Poisson equation," *Journal of Physics A: Mathematical and Theoretical*, vol. 46 (2013), 115201.

Samples of minimal polynomials found by multipair PSLQ

s	Minimal polynomial corresponding to $x = y = 1/s$:
5	$1 + 52\alpha - 26\alpha^2 - 12\alpha^3 + \alpha^4$
6	$1 - 28\alpha + 6\alpha^2 - 28\alpha^3 + \alpha^4$
7	$-1 - 196\alpha + 1302\alpha^2 - 14756\alpha^3 + 15673\alpha^4 + 42168\alpha^5 - 111916\alpha^6 + 82264\alpha^7$ $-35231\alpha^8 + 19852\alpha^9 - 2954\alpha^{10} - 308\alpha^{11} + 7\alpha^{12}$
8	$1 - 88\alpha + 92\alpha^2 - 872\alpha^3 + 1990\alpha^4 - 872\alpha^5 + 92\alpha^6 - 88\alpha^7 + \alpha^8$
9	$-1 - 534\alpha + 10923\alpha^2 - 342864\alpha^3 + 2304684\alpha^4 - 7820712\alpha^5 + 13729068\alpha^6$ $-22321584\alpha^7 + 39775986\alpha^8 - 44431044\alpha^9 + 19899882\alpha^{10} + 3546576\alpha^{11}$ $-8458020\alpha^{12} + 4009176\alpha^{13} - 273348\alpha^{14} + 121392\alpha^{15}$ $-11385\alpha^{16} - 342\alpha^{17} + 3\alpha^{18}$
10	$1 - 216\alpha + 860\alpha^2 - 744\alpha^3 + 454\alpha^4 - 744\alpha^5 + 860\alpha^6 - 216\alpha^7 + \alpha^8$

These computations are very expensive. The case $x = y = 1/32$, for instance, required 10,000-digit arithmetic and ran for 45 hours. Other runs, using even higher precision, ultimately failed, evidently due to subtle program bugs. Help!

Kimberley's formula for the degree of the polynomial

Based on our preliminary results, Jason Kimberley of the University of Newcastle, Australia observed that the degree $m(s)$ of the minimal polynomial associated with the case $x = y = 1/s$ appears to be given by the following:

Set $m(2) = 1/2$. Otherwise for primes p congruent to 1 mod 4, set $m(p) = \mathrm{int}^2(p/2)$, where int denotes greatest integer, and for primes p congruent to 3 mod 4, set $m(p) = \mathrm{int}\,(p/2)(\mathrm{int}\,(p/2) + 1)$. Then for any other positive integer s whose prime factorization is $s = p_1^{e_1} p_2^{e_2} \cdots p_r^{e_r}$,

$$m(s) \stackrel{?}{=} 4^{r-1} \prod_{i=1}^{r} p_i^{2(e_i-1)}\, m(p_i).$$

Does Kimberley's formula hold for larger s? Why?

What is the true mathematical connection between the pair of rationals (x, y) and the algebraic number α?

Three improvements to the Poisson polynomial computation program

1. MPFUN2015: A new thread-safe multiprecision package.
   - Speedup: 3X
2. A new 3-level multipair PSLQ program.
   - Speedup: 4.2X
3. Parallel implementation on a 16-core system.
   - Speedup: 12.2X

Overall speedup: 156X

Thread safety in high-precision software

Even though parallel implementations are often required for high-precision computations, most high-precision software packages are not thread-safe and thus cannot be used in shared memory parallel programs.

- Many packages employ global read/write variables, e.g., for transcendental function evaluation, which ruin thread safety.
- The working precision level, a global variable that is changed frequently within the package itself and often by users also, is particularly troublesome.

One bright spot: the MPFR package

- Thread-safe (if compiled with the thread-safe option).
- Correct rounding to the last bit.
- Fastest package currently available.
- Based on the Gnu multiprecision package (GMP).
- Low-level arithmetic and transcendental functions only.

MPFUN2015: DHB's thread-safe arbitrary precision package

Available in two versions:

- MPFUN-Fort: Completely self-contained, all-Fortran version. Compilation is a simple one-line command, which completes in a few seconds.
- MPFUN-MPFR: Calls the MPFR package for lower-level operations. Installation is significantly more complicated (since GMP and MPFR must first be installed), but performance is roughly 3X faster. We used MPFUN-MPFR in this study.

Both versions include a high-level language interface, using custom datatypes and operator overloading — for most applications, only a few minor changes to existing double precision code are required.

Designed for high-level Fortran programs; a C++ version is planned but not written.

- D. H. Bailey, "MPFUN2015: A thread-safe arbitrary precision computation package," manuscript, `http://www.davidhbailey.com/dhbpapers/mpfun2015.pdf`.

- Software is available at `http://www.davidhbailey.com/dhbsoftware`.

New three-level multipair PSLQ program

Employs three levels of numeric precision:

- Ordinary double precision.
- Medium precision, typically 100–2000 digits.
- Full precision, typically many thousands of digits.

When an entry of the double precision reduced vector is smaller than 10^{-14}, the medium precision arrays are updated by matrix multiplication.

Similarly, when an entry of the medium precision reduced vector is smaller than the medium precision "epsilon," the full-precision arrays are updated by matrix multiplication.

Substantial care must be taken to manage this three-level hierarchy, and to correctly handle numerous atypical scenarios.

Jon Borwein's fast algorithm to compute $\phi_2(x, y)$

$$\phi_2(x, y) = \frac{1}{2\pi} \log \left| \frac{\theta_2(z, q)\theta_4(z, q)}{\theta_1(z, q)\theta_3(z, q)} \right|,$$

where $q = e^{-\pi}$ and $z = \frac{\pi}{2}(y + ix)$. Compute the four theta functions using the following very rapidly convergent formulas involving complex variables:

$$\theta_1(z, q) = 2 \sum_{k=1}^{\infty} (-1)^{k-1} q^{(2k-1)^2/4} \sin((2k-1)z),$$

$$\theta_2(z, q) = 2 \sum_{k=1}^{\infty} q^{(2k-1)^2/4} \cos((2k-1)z),$$

$$\theta_3(z, q) = 1 + 2 \sum_{k=1}^{\infty} q^{k^2} \cos(2kz),$$

$$\theta_4(z, q) = 1 + 2 \sum_{k=1}^{\infty} (-1)^k q^{k^2} \cos(2kz).$$

High-level computational algorithm

1. Given rationals $x = p/s$ and $y = q/s$, select a conjectured minimal polynomial degree $m(s)$ (using Kimberley's formula) and other parameters for the run.
2. Calculate $\phi_2(x, y)$ to P_2-digit precision using the formulas from two viewgraphs above. When done, calculate $\alpha = \exp(8\pi\phi_2(x, y))$ and generate the $(m + 1)$-long vector $X = (1, \alpha, \alpha^2, \cdots, \alpha^m)$, to P_2-digit precision.
3. Apply the three-level multipair PSLQ algorithm to X. For larger problems, employ a parallel version of the three-level multipair PSLQ code, using the OpenMP construct `DO PARALLEL` to perform certain time-intensive loops in parallel.
4. If no numerically significant relation is found, try again with a larger degree m or higher precision P_2. If a relation is found, employ the polynomial factorization facilities in *Mathematica* and *Maple* to ensure that the polynomial is irreducible.

Application program and libraries for the Poisson calculations

Description	Language	Lines of code
Poisson polynomial program*	Fortran	2,000
MPFUN-MPFR package	Fortran	12,000
MPFR package	C	93,000
GMP package	C	83,000
Total		190,000

*This includes the computation of $\phi_2(x, y)$ and the 3-level multipair PSLQ program.

192-degree minimal polynomial found by multipair PSLQ for $x = y = 1/35$

This distinctive semi-elliptical pattern is strong evidence that the result is correct.

Timings for the case $x = y = 1/35$

Multiprecision software	PSLQ code	Cores	Run time	Speedup
ARPREC	2-level	1	$1.599 \cdot 10^6$	1.00
MPFUN-MPFR	2-level	1	$5.249 \cdot 10^5$	3.05
MPFUN-MPFR	3-level	1	$1.240 \cdot 10^5$	12.90
		2	$7.585 \cdot 10^4$	21.08
		4	$4.121 \cdot 10^4$	38.80
		8	$2.476 \cdot 10^4$	64.58
		16	$1.021 \cdot 10^4$	156.61

The run times are wall-clock run times (in seconds), measured on a 16-core 2.4 GHz MacPro, in a typically busy environment with similar jobs running on other cores.

Selected runs (degrees, precision, timings, etc.) for $x = y = 1/s$

s	m	$\log_{10}(D)$	P_1	P_2	N	M	C	T (sec.)	$C \cdot T$ (sec.)
20	32	-463.84	160	700	1967	0.81	1	$2.19 \cdot 10^0$	$2.19 \cdot 10^0$
24	64	-1883.78	320	2200	9297	11.33	1	$7.73 \cdot 10^1$	$7.73 \cdot 10^1$
30	64	-1868.01	350	2300	9064	11.33	1	$1.02 \cdot 10^2$	$1.02 \cdot 10^2$
32	128	-7577.07	650	8200	45893	168.20	1	$5.13 \cdot 10^3$	$5.13 \cdot 10^3$
34	128	-7574.93	650	8200	45914	168.20	1	$5.16 \cdot 10^3$	$5.16 \cdot 10^3$
36	144	-9570.86	750	10300	62282	267.10	1	$9.54 \cdot 10^3$	$9.54 \cdot 10^3$
38	180	-14951.64	900	16000	120984	642.98	1	$3.88 \cdot 10^4$	$3.88 \cdot 10^4$
40	128	-7580.00	650	8200	45655	168.20	1	$5.02 \cdot 10^3$	$5.02 \cdot 10^3$
42	192	-16993.99	1000	18000	150364	829.41	8	$1.57 \cdot 10^4$	$1.26 \cdot 10^5$
44	240	-26604.14	1200	28000	323762	2003.33	8	$7.43 \cdot 10^4$	$5.94 \cdot 10^5$
46	264	-32036.34	1350	34000	476902	2921.57	16	$1.06 \cdot 10^5$	$1.70 \cdot 10^6$
48	256	-30248.55	1350	32000	415316	2586.39	16	$8.98 \cdot 10^4$	$1.44 \cdot 10^6$
50	200	-18421.18	1000	20000	168947	974.44	8	$2.12 \cdot 10^4$	$1.69 \cdot 10^5$
52	288	-38414.49	1550	41000	655291	4124.24	16	$2.12 \cdot 10^5$	$3.40 \cdot 10^6$
*60	256	-14477.99	800	16000	90371	336.41	1	$5.28 \cdot 10^3$	$5.28 \cdot 10^3$
*64	512	-57816.90	1600	64000	802361	5172.79	16	$3.78 \cdot 10^5$	$2.42 \cdot 10^6$

s = denominator; m = degree; D = detection level; P_1 = medium precision; P_2 = full precision; N = number of iterations; M = Mbytes; C = cores; T = wall clock time; $C \cdot T$ = total core-seconds.

Palindromic polynomials

From our results, in the case $(1/s, 1/s)$ where s is even, the resulting polynomial appears to be palindromic ($a_k = a_{m-k}$). For instance, when $s = 16$,

$$\begin{aligned} p_{16}(\alpha) = 1 &- 1376\alpha^1 - 12560\alpha^2 - 3550496\alpha^3 + 81241720\alpha^4 - 169589984\alpha^5 \\ &+ 1334964944\alpha^6 - 24307725984\alpha^7 + 238934926108\alpha^8 - 1043027124704\alpha^9 \\ &+ 2328675366384\alpha^{10} - 3219896325280\alpha^{11} + 4238551472456\alpha^{12} \\ &- 10247414430048\alpha^{13} + 28552105805904\alpha^{14} - 55832851687968\alpha^{15} \\ &+ 70020268309062\alpha^{16} \\ &- 55832851687968\alpha^{17} + 28552105805904\alpha^{18} - 10247414430048\alpha^{19} \\ &+ 4238551472456\alpha^{20} - 3219896325280\alpha^{21} + 2328675366384\alpha^{22} \\ &- 1043027124704\alpha^{23} + 238934926108\alpha^{24} - 24307725984\alpha^{25} + 1334964944\alpha^{26} \\ &- 169589984\alpha^{27} + 81241720\alpha^{28} - 3550496\alpha^{29} - 12560\alpha^{30} - 1376\alpha^{31} + \alpha^{32} \end{aligned}$$

Nitya Mani, an undergraduate student at Stanford University, observed that if α is a root of a palindromic polynomial such as this, then $\alpha + 1/\alpha$ is a root of a transformed polynomial of half the degree. This fact can be used to significantly accelerate the computation of Poisson polynomials in the even case (runs denoted by * in the table).

New observations for the case $(1/s, 1/s)$

After doing some Google searches on the coefficients of the polynomials p_{11} and p_{13}, we found the coefficient 387221579866 in p_{11} appears in a 2010 preprint by Savin and Quarfoot, and the coefficient 221753896032 in p_{13} appears in a manuscript, also dated 2010, by Bostan, Boukraa, Hassani, Maillard, Weil, Zenine and Abarenkova.

Savin and Quarfoot define a sequence ψ_n of polynomials in x and y, based on the curve $y^2 = x^3 + x$, as follows:

$$\begin{aligned} \psi_1 &= 1 \\ \psi_2 &= 2y \\ \psi_3 &= 3x^4 + 6x^2 - 1 \\ \psi_4 &= 2y(2x^6 + 10x^4 - 10x^2 - 2), \end{aligned} \tag{1}$$

and, recursively,

$$\begin{aligned} \psi_{2n+1} &= \psi_{n+2} \cdot \psi_n^3 - \psi_{n-1} \cdot \psi_{n+1}^3 \quad \text{for } n \geq 2 \\ \psi_{2n} &= 1/(2y) \cdot \psi_n(\psi_{n+2} \cdot \psi_{n-1}^2 - \psi_{n-2} \cdot \psi_{n+1}^2) \quad \text{for } n \geq 3. \end{aligned}$$

Our analysis

We constructed a related sequence J_s of integer coefficient polynomials in a by setting $x = \sqrt{-a}$, and so $y^2 = x(x^2+1) = \sqrt{-a}(1-a)$; we also remove the leading $2y$:

$$\begin{aligned} J_{2n+1}(a) &= \psi_{2n+1}(x,y) \\ J_{2n}(a) &= 1/(2y) \cdot \psi_{2n}(x,y) \end{aligned}$$

The initial values of $J_s(a)$ are

$$\begin{aligned} J_1 &= 1 \\ J_2 &= 1 \\ J_3 &= 3a^2 - 6a - 1 \\ J_4 &= 2a^3 - 10a^2 - 10a + 2. \end{aligned}$$

After computation in *Magma*, we were able to prove that for each prime $q \equiv 3 \bmod 4$ the polynomial J_q has degree $m(q)$, where $m(s)$ is Kimberley's formula.

Additional conjectures

In fact, our computations support these conjectures:

- For each prime $q \equiv 3 \bmod 4$, the polynomial J_q is precisely p_q as computed by PSLQ.
- For each integer $s \geq 1$, p_s is the unique degree $m(s)$ prime factor of J_s.
- The J function is a divisibility sequence: $m \mid n$ implies $J_m \mid J_n$.
- For each positive integer s, both J_s and p_s have largest real root α_s.

Proofs of Kimberley's formula and the palindromic property

- In March 2016, DHB presented our results at a seminar at the University of California, Berkeley.
- Following the presentation, Watson Ladd, a graduate student in mathematics, brought to our attention the fact that some of our conjectures should follow from results in the theory of elliptic curves, Gaussian integers and ideals.
- After some effort, Ladd produced proofs of Kimberley's formula and the palindromic property, which proofs were then incuded in our paper and returned to the journal.

Poisson polynomials: Progress and challenges

- Our computations employed up to 64,000-digit precision, producing polynomials with degrees up to 512 and integer coefficients up to 10^{229}. These are the largest successful integer relation computations to date.
- Kimberley's formula was affirmed in every case $x = y = 1/s$, for s up to 52 (except for $s = 41, 43, 47, 49, 51$), and also for $s = 60$ and $s = 64$.
- The resulting polynomial coefficients yielded clues that ultimately led to a proof of Kimberley's formula and the palindromic property, employing techniques of elliptic curves, Gaussian integers and ideals.
- Additional research is needed to understand many other combinations, e.g., $x = p/s$ and $y = q/s$, for different values of p, q and s.
- A fundamentally new integer relation algorithm may be required to further extend the requisite computations.

Full details are at:
http://www.davidhbailey.com/dhbpapers/poisson-res.pdf

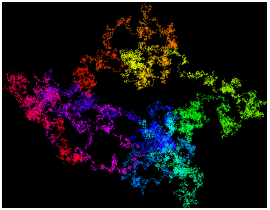
Jon Borwein's research on π and experimental mathematics

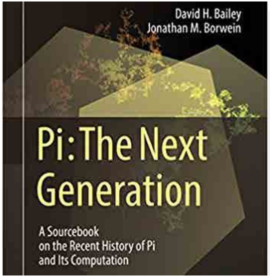
Jon's interest in π and experimental math was prompted in part by his desire to do research that would connect to a large public audience.

π continues to excite millions worldwide, leading many to pursue careers in math, science and engineering.

Experimental mathematics enables a much broader community to do real math research:

- High school and college students.
- Computer scientists.
- Computer graphics experts.
- Statisticians.
- Data scientists.





Von Neumann's warning about the future of mathematics

Experimental mathematics provides a means to escape the trap feared by John von Neumann when he wrote,

> *But there is a grave danger that the subject [of mathematics] will develop along the line of least resistance, that the stream, so far from its source, will separate into a multitude of insignificant branches, and that the discipline will become a disorganized mass of details and complexities. In other words, at a great distance from its empirical source, or after much "abstract" inbreeding, a mathematical subject is in danger of degeneration. . . .*
>
> *In any event, whenever this stage is reached, the only remedy seems to me to be the rejuvenating return to the source: the re-injection of more or less directly empirical ideas. I am convinced that this was a necessary condition to conserve the freshness and the vitality of the subject and that this will remain equally true in the future.*

Winning the battle, but losing the war

Mathematicians and scientists may be winning battles to publish papers and obtain grants, but we are losing the war for the hearts and minds of the public:

- 51% in U.S. (54% in Australia) either do not believe in climate change, or do not believe there is any human connection.
- 42% in U.S. (23% in Australia) believe that humans were created within past 10,000 years.
- 38% in U.S. (32% in Australia) do not believe in evolution.
- 32% in U.S. (24% in Australia) do not believe vaccinations are safe.
- 48% in U.S. (34% in Australia) believe humans are being visited by extraterrestrial UFOs.
- 6% in U.S. believe NASA faked the Apollo moon landings.
- Some even dispute the value of π. (I frequently receive such email.)

Anti-science movements arise from both sides of the political spectrum:

- From the left: anti-vaccination and anti-fluoridation.
- From the right: anti-climate change and anti-evolution.

Carl Sagan's warning (*The Demon Haunted World*, 1995)

I have a foreboding of an America in my children's or my grandchildren's time — when the United States is a service and information economy; when nearly all the key manufacturing industries have slipped away to other countries; when awesome technological powers are in the hands of a very few, and no one representing the public interest can even grasp the issues; when the people have lost the ability to set their own agendas or knowledgeably question those in authority; when, clutching our crystals and nervously consulting our horoscopes, our critical faculties in decline, unable to distinguish between what feels good and what's true, we slide, almost without noticing, back into superstition and darkness. ...

We've arranged a global civilization in which most crucial elements ... profoundly depend on science and technology. We have also arranged things so that almost no one understands science and technology. This is a prescription for disaster. We might get away with it for a while, but sooner or later this combustible mixture of ignorance and power is going to blow up in our faces.

How can we turn the tide?

- Start a blog.
- Visit schools or give lectures.
- Write books for the general public.
- Write articles for science news forums.
- Write expository articles for scientific journals.
- Pursue research topics that have potentially wide appeal.
- Recognize communication skills in hiring, promotion and research grant decisions.
- Find ways to utilize computers and otherwise make teaching and research much more engaging and interesting.

Math Drudge

Two mathematicians contemplate the cosmos

NEW MATH SCHOLAR BLOG | EXPERIMENTAL MATH SITE | JON BORWEIN MEMORIAL SITE | FINANCIAL MATH SITE

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« Where is ET? Fermi's paradox turns 65 Is the nature of mathematical proof changing

I Prefer Pi: Background for Big Pi Day (3/14/15)

"I Prefer Pi" is appropriate title for Pi Day (3/14, i.e., March 14), as it is one of the few palindromes involv
Pi = 3.141592653589793... (a palindrome is a phrase that reads the same forwards or backwards).

Pi Day is particularly memorable this year, since only once a century can one celebrate this event in a year where the longer version 3/14/15 continues two more correct digits of Pi. The Museum of Mathematics in New York City, among others, is taking Pi Day 2015 one step further, by celebrating at 9:26am, i.e., 3/14/15/926, adding three more digits. See MoMath's website for details.

Chicagoans plan to celebrate by running in a Pi-K race of 3.14 miles. Numerous city bakeries are offering special pies for the occasion at $3.14 per slice.

Not as well known perhaps is the fact that March 14 is also the 136th birthday of Albert Einstein, and that 2015 is the 100th anniversary of the publication of Einstein's paper on general relativity. To commemorate this year's doubly significant event, Princeton University is planning a gala event, including a pie eating contest, a performance by the Princeton Symphony, a contest to see who can recite the most correct digits of pi, and a guided Einstein tour.

Pi in the popular culture

Pi Day long ago extended its reach beyond a handful of mathematical zealots, to become a widely celebrated event. For example, the March 14, 2007 New York Times crossword puzzle featured clues, where, in numerous locations, a pi character (standing for PI) must be entered at the intersection of two words. For example, 33 across "Vice president after Hubert" (answer: SPIRO) intersects with 34 down "Stove feature" (answer: PILOT). Indeed 28 down, with clue "March 14, to mathematicians," was, appropriately enough, PIDAY, while PIPPIN is now a four-letter word.

In 2009, the U.S. House of Representatives passed resolution officially designating March 15 as as "National Pi Day," and encouraging "schools and educators to observe th
day with appropriate activities that teach students about Pi and engage them about the study of mathematics." This may well be the first legislation on Pi Day to have been adopted by a national governmental body.

In general, pi is much more in the public eye than it was even five or ten years ago. On May 9, 2013, the North American quiz show Jeopardy! featured an entire category of
questions on pi. The clues provided were:

1. ($200) Pi is the ratio of this measurement of a circle to its diameter.
2. ($400) Numerically, pi is considered this, like a type of "meditation".
3. ($600) For about $19,100 x pi, this "Black Swan" director made "Pi", his 1998 debut film about a math whiz.
4. ($800) In the 100s A.D. this Alexandrian astronomer calculated a more precise value of pi, the equivalent of 3.14166.
5. ($1000) You can find the area of this oval geometric shape with pi x A x B, if A & B are half of its longest & shortest diameter.

The clues and the answers (all were answered correctly by various contestants) are given here in the J-archive, an independent repository of clues and answers maintained by Jeopardy! fans.

Some other recent examples of the public's mania for pi include the following:

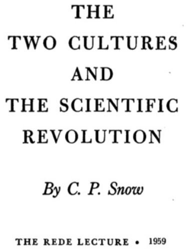
1. On September 12, 2012, five aircraft armed with dot-matrix-style skywriting technology wrote 1000 digits of pi in the sky above the San Francisco Bay Area as a spectacular if costly piece of *piformance* art.
2. On March 14, 2012 (appropriately enough), U.S. District Court Judge Michael H. Simon dismissed a copyright infringement suit relating to the lyrics of a song by rulin
that "Pi is a non-copyrightable fact."
3. On August 18, 2005, Google offered 14,159,265 "new slices of rich technology" in their initial public stock offering. On January 29, 2013 they offered a pi-million doll
prize for successful hacking of the Chrome Operating System on a specific Android phone.

Ending the war between science and the humanities

Given the growing tensions in society, and the impact of rapidly changing technology, we can no longer afford a war between the science-tech world and the humanities:

- Those in math, science and technology must learn more about the humanities, to better appreciate these fields, and to better communicate to the public.
- Those in the humanities must learn more about math, science and technology, to better appreciate these fields, and to better participate in dialogue on key issues.

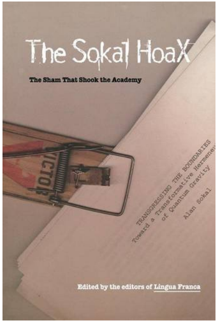
It's in Apple's DNA that technology alone is not enough — that it's technology married with liberal arts, married with the humanities, that yields us the result that makes our hearts sing. [Steve Jobs]



THE
TWO CULTURES
AND
THE SCIENTIFIC
REVOLUTION

By C. P. Snow

THE REDE LECTURE • 1959



They should have sent a poet

In one memorable scene from the movie *Contact*, Jodi Foster views a galaxy from her spacecraft, and is so overcome with awe that she exclaims,

> They should have sent a poet. So beautiful.
> So beautiful... I had no idea.

In a similar way, those of us involved in research are often stunned by the beauty and elegance of mathematics and science, along with the rather mysterious fact that we humans are able to comprehend these laws.

So why don't we do more to share this wonder? Why don't we write some poetry?

Thanks!
This talk is available here: `http://www.davidhbailey.com/dhbtalks/dhb-jbcc-2017.pdf`