Experience using AI software to prove Euler sum results

David H. Bailey
Lawrence Berkeley National Laboratory
dhbailey@lbl.gov

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1   Introduction

Recently the rise of large language models and other AI software systems has attracted considerable attention. Several leading mathematicians, among others, have commented on their progress. In 2024, for instance, Terence Tao predicted the following [6]:

> In three years you will see notable progress, and it will become more and more manageable to actually use AI. ... And so with AI, we can start proving hundreds of theorems or thousands of theorems at a time. And human mathematicians will direct the AIs to do various things.

More recently, in February 2026, Tao updated his assessment in an *Atlantic* article [8]:

> There have been more sophisticated solutions, which are AI-assisted. I think in the short term we're going to get a lot of quick wins on easy problems from pure AI methods. And then over the next few months, I think we're going to have all kinds of hybrid, human-AI contributions.

Some tests of mathematical proficiency have gone quite well (see for instance [5]), but others have not. To better measure this progress, in February 2026 a consortium of researchers presented a set of ten research-level mathematical questions to be posed to currently available AI agents [1, 7]. These researchers are currently assessing the performance of various systems on these problems.

In February 2023, the present author requested ChatGPT to provide proofs to four well-known mathematical theorems (briefly, the impossibility of trisection, the irrationality of π, the transcendence of π, and the fundamental theorem of algebra) [2]. The results were not very impressive: for example, ChatGPT attempted to prove the irrationality of π by reasoning (circularly) that if π were rational, then its decimal expansion would eventually be periodic, but since "it has been proven that the decimal representation of π is non-repeating and non-terminating," then π is irrational (the non-repeating nature of the digits of π is a consequence of its irrationality, not the other way around).

In February 2025, the author revisited this exercise [3], this time using DeepSeek, which had recently been released. DeepSeek did fairly well, presenting coherent if somewhat terse proofs, nicely typeset, for three of the four requested theorems. The author noted a few nits, and would have preferred more details and references, but in general DeepSeek's performance appeared promising.

One year later, given the very rapid improvement in these models, it is time to assess their ability to handle some more realistic research problems. To that end, the author offers this modest study of four software systems on some research problems from the theory of Euler sums.

2   Euler sum problems

An *Euler sum* is an infinite series involving the harmonic function $H(k) = 1 + 1/2 + 1/3 + \cdots + 1/k$. They have been studied since the time of Euler, and more recently in numerous investigations of mathematical

physics, the Riemann hypothesis and others. One notable feature of these sums is that many have surprisingly elegant analytic evaluations: for example, $\sum_{k=1}^{\infty} H(k)/k^2 = 2\zeta(3)$, $\sum_{k=1}^{\infty} H(k)/k^3 = \pi^4/72$ and $\sum_{k=1}^{\infty} H(k)^2/k^3 = 7\zeta(5)/2 - \pi^2\zeta(3)/6$, where $\zeta(\cdot)$ is the Riemann zeta function.

Recently the present author, in collaboration with Ross McPhedran (University of Sydney, Australia) and Bruno Salvy (INRIA, France) found some formulas and techniques giving explicit evaluations, in terms of the polygamma function, for a class of Euler sums. We originally discovered these results numerically, then subsequently found rigorous proofs for them. While these results were not trivial, they certainly would not be considered "hard" mathematical problems. They are more in the category of problems that could be assigned to a good student as part of a larger research effort. These proofs are in a manuscript [4] that we have not yet made publicly available (in part to avoid negating this study with online material), and, by the way, employed an approach distinct from any mentioned below.

In the course of this research, the present author tried posing some of these questions to several currently available large language models, including ChatGPT, DeepSeek, Google's Gemini and Anthropic's Claude. This exercise was an interesting exploration into both the capabilities and limitations of these systems. Some specific problems that the present author has posed to these models include the following, ranging from general (more challenging) to specific (less challenging).

Notation. In the following, $H(k) = H_k = 1 + 1/2 + \cdots + 1/k$ denotes the harmonic function; $\psi(q, z) = \psi^{(q)}(z) = D^{q+1}(\log \Gamma(z))$ denotes the polygamma function (here D^{q+1} is the $(q+1)$-th derivative); $\psi(z) = \psi(0, z)$ is the digamma function; the notation $\binom{n}{k}$ denotes the binomial coefficient; and $\gamma = 0.5772156649\ldots$ is Euler's constant.

Problems:

1. Given t not an integer, and integer $p \geq 2$, prove that

$$\sum_{k=1}^{\infty} \frac{H(k)}{(k+t)^p} = \frac{1}{2(p-1)!} \left(t^{-p-1}p! + (-1)^p 2(\gamma + \psi(0,t))\psi(p-1,t) \right.$$
$$\left. + (-1)^p \sum_{k=1}^{p-2} \binom{p-1}{k} \psi(k,t)\psi(p-1-k,t) - (-1)^p \psi(p,1+t) \right). \qquad (1)$$

2. For integers $m, n \geq 1, \gcd(m,n) = 1, p \geq 2$, prove that

$$\sum_{k=1}^{\infty} \frac{H(k)}{(mk+n)^p} = \frac{1}{2m^p(p-1)!} \left((n/m)^{-p-1} p! + (-1)^p 2(\gamma + \psi(0, n/m))\psi(p-1, n/m) \right.$$
$$\left. + (-1)^p \sum_{k=1}^{p-2} \binom{p-1}{k} \psi(k, n/m)\psi(p-1-k, n/m) - (-1)^p \psi(p, 1+n/m) \right). \qquad (2)$$

3. Prove that

$$\sum_{k=1}^{\infty} \frac{H(k)}{(5k+1)^2} = \frac{1}{50} \left(250 + 2\gamma\psi(1, 1/5) + 2\psi(0, 1/5)\psi(1, 1/5) - \psi(2, 6/5) \right). \qquad (3)$$

The present author did not perform a comprehensive check of all problems and models, but here are a few examples of the results obtained, mostly for Problem 3, the simplest in the list.

3 ChatGPT

The author first tried ChatGPT, in particular the version available 10 February 2026 on the website https://www.chatgpt.com. The typed prompt was merely a statement of Problem 3 above:

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Please prove that the sum of harmonic number of k divided by (5*k+1)^2, for k=1 to infinity,
equals 5 + 1/25*Euler's constant*polygamma(1,1/5) + 1/25*polygamma(0,1/5)*polygamma(1,1/5)
- 1/50*polygamma(2,6/5).
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NOTE: In the following, and in the discussion of other software below, the author presents the formulas produced by the software in the typeset style employed by the software, even though in some cases it differs from that of other software, and also often differs from the author's preferred style. The full output produced by the software has been retained by the author and is available if desired.

Analysis: At first glance, the present author was impressed by the result, as it was clear that ChatGPT had correctly transcribed and comprehended the problem.

In its Step 1, ChatGPT cleverly employed the identity $H_k = \int_0^1 (1 - x^k)/(1 - x)\, dx$ to rewrite the original summation on the left-hand side of (3) as $\int_0^1 1/(1-x) \sum_{k=1}^\infty (1-x^k)/(5k+1)^2\, dx$. This interchange of summation and integration is valid, but the present author was disappointed with the brief justification "absolute convergence holds," as well as the lack of a cited reference for the identity. Next, in its Step 2, ChatGPT employed the identity $\Phi(z, s, a) = \sum_{k=0}^\infty z^k/(k + a)^s$, where $\Phi(z, s, a)$ denotes the Lerch phi function, to write the line

$$\sum_{k=1}^\infty \frac{x^k}{(5k + 1)^2} = \frac{1}{25} \sum_{k=1}^\infty \frac{(x^5)^k}{(k + \frac{1}{5})^2} = \frac{1}{25} \left[\Phi(x^5, 2, 1/5) - \frac{1}{(1/5)^2} \right]. \tag{4}$$

Sadly, the middle expression is completely in error — the numerator $(x^5)^k$ should simply be x^k, and thus the right-hand expression is not equal to the left-hand summation.

ChatGPT continues this line of reasoning in its Steps 3 and 4. While there is no need to dissect this material, since it is in error from Step 2 onward, it is worth mentioning that near the end of Step 4 ChatGPT employed a "known identity (derivable by differentiating Hurwitz zeta)":

$$\sum_{n=0}^\infty \frac{\psi(n + b)}{(n + a)^2} = \psi(a)\psi^{(1)}(a) - \frac{1}{2}\psi^{(2)}(a + b - a). \tag{5}$$

Sadly, this identity, for which no reference is provided, is false: note for arguments $a = b = 1/2$, the left-hand side evaluates to $-7.0556442944\ldots$, while the right-hand side evaluates to $-1.2751352750\ldots$. ChatGPT subsequently produced the line $I = 25 + \gamma\psi^{(1)}(\frac{1}{5}) + \psi(\frac{1}{5})\psi^{(1)}(\frac{1}{5}) - \frac{1}{2}\psi^{(2)}(\frac{6}{5})$, which, after dividing by 25, it claimed to yield "exactly" formula (3). But this is also false — after dividing the expression I by 25, the initial coefficient would be 1, not 5.

The present author then tried a more advanced version of ChatGPT, namely GPT5.1, available from the site https://use.ai on 17 February 2026 (this required a paid subscription). After the initial output was complete, the author selected the option to have the tool produce fully detailed output.

At first glance GPT5.1's output looked promising. But on closer inspection, several problems were evident. Its strategy revolved around the identity

$$\sum_{k=1}^\infty \frac{H_k}{(5k + 1)^2} = \sum_{n=0}^\infty \sum_{k=1}^\infty \frac{1}{(5k + 1)^2} \left(\frac{1}{n + 1} - \frac{1}{n + k + 1} \right). \tag{6}$$

So far, so good. However, GPT5.1 then pursued an argument that in one section produced several equations with divergent sums, such as the enigmatic line

$$\sum_{k=1}^\infty \frac{1}{k + a} = \sum_{k=0}^\infty \frac{1}{k + a + 1} = \sum_{k=0}^\infty \frac{1}{k + (a + 1)}. \tag{7}$$

It subsequently argued, among other things, that

$$\sum_{k=1}^{\infty} \frac{1}{(5k+1)(k+a)} = \frac{1}{5a-1} \sum_{k=1}^{\infty} \left(\frac{1}{k+a} - \frac{5}{5k+1} \right) = \frac{1}{5a-1} \left(-\psi(a+1) + \psi\left(\frac{1}{5}\right) + 5 \right)$$

$$\sum_{k=1}^{\infty} \frac{1}{(5k+a)(k+b)} = \frac{1}{5b-a} \sum_{k=1}^{\infty} \left(\frac{1}{k+b} - \frac{5}{5k+a} \right) = \frac{1}{5b-a} \left[-\psi(b+1) + \psi\left(\frac{a}{5}\right) + \frac{5}{a} \right]. \quad (8)$$

In each line, the second and third portions have a sign error, compared with the first. From here GPT5.1 attempted to reach the desired conclusion, after a lengthy derivation, which included several additional instances of divergent sums. In short, while GPT5.1's approach was interesting, the output was too problematic to be useful.

For both the versions of this software, ChatGPT's claim that it had produced "exactly" the desired result is quite concerning. If it had not been able to derive the requested result according to well-established standards of mathematical rigor, it would have been much more useful to graciously acknowledge this fact to the user, rather than to offer unreliable material.

4 DeepSeek

The author then tried DeepSeek, in particular the version available 10 February 2026 on the website https://www.deepseek.com. The problem posed to DeepSeek is exactly the same as was posed to ChatGPT.

Analysis: Just as with ChatGPT, at first glance the DeepSeek output seemed impressive — it was nicely typeset (but with some glitches), and it is clear that DeepSeek understood the problem.

In Step 1, DeepSeek first employed the identity $H_k = \psi^{(0)}(k+1) + \gamma$ to decompose the original sum into two parts, then employed the identity $\psi^{(1)}(a) = \sum_{k=0}^{\infty} 1/(k+a)^2$ to evaluate the second (easier) sum. Then beginning in Step 3, it employed the identity $H_k = \int_0^1 (1-x^k)/(1-x)\,dx$, without a reference citation, to (correctly) rewrite the original sum S as

$$S = \sum_{k=1}^{\infty} \frac{H_k}{(5k+1)^2} = \int_0^1 \frac{dx}{1-x} \sum_{k=1}^{\infty} \frac{1-x^k}{(5k+1)^2}. \quad (9)$$

After some additional manipulation, in Step 5 it employed the Lerch phi function, with some success. Beginning in Step 6, however, DeepSeek went awry with the line

$$S = \int_0^1 \frac{1}{1-x}dx + \frac{1}{25} \int_0^1 \frac{\psi^{(1)}(1/5) - 1 - \Phi(x, 2, 1/5)}{1-x}\,dx. \quad (10)$$

DeepSeek acknowledged that the first integral diverges, but it dismissed the problem with the comment "the pole at $x = 1$ cancels between S_2 and Φ term," a terse explanation that would not be acceptable in professional work. DeepSeek then cited a "known formula":

$$\sum_{k=1}^{\infty} \frac{H_k}{(k+a)^2} = \frac{1}{a} \left[\psi^{(0)}(a+1)\psi^{(1)}(a) - \frac{1}{2}\psi^{(2)}(a+1) \right] + \frac{\psi^{(1)}(a) - 1/a}{a^2} + \frac{\gamma\psi^{(1)}(a)}{a} + \frac{\psi^{(0)}(a+1)}{a^2}. \quad (11)$$

Sadly, this formula is false: note if one sets $a = 1/2$, for instance, the left-hand side summation evaluates to $7\zeta(3) - \pi^2 \log(2) = 1.57330985826\ldots$, whereas the right-hand side evaluates to $18.7709972167\ldots$.

DeepSeek then concluded with some additional discussion, before finally declaring the completion of its proof. So in short, while DeepSeek definitely tried some interesting approaches, the lengthy output included invalid reasoning and thus was not reliable.

5 Google Gemini

The author then tried Google Gemini, in particular the version available 17 February 2026 on the website https://gemini.google.com. The problem posed is exactly the same as was posed to ChatGPT.

Analysis: As with ChatGPT and DeepSeek, Gemini's output looked fairly professional, with nicely typeset mathematical notation. It correctly comprehended the problem.

In Step 1, Gemini employed an approach, which by the way was also tried by both ChatGPT and DeepSeek, namely to rewrite the original summation as

$$\sum_{k=1}^{\infty} \frac{H_k}{(5k+1)^2} = \sum_{k=1}^{\infty} \frac{1}{(5k+1)^2} \int_0^1 \frac{1-x^k}{1-x} \, dx = \int_0^1 \frac{1}{1-x} \sum_{k=1}^{\infty} \left(\frac{1}{(5k+1)^2} - \frac{x^k}{(5k+1)^2} \right) \, dx, \quad (12)$$

although it did not justify this interchange of summation and integration.

The proof continued until Step 3, where it presented a "known powerful identity"

$$\sum_{k=1}^{\infty} \frac{H_k}{(k+a)^2} = \frac{1}{2} \left[\psi^{(2)}(a+1) + 2\gamma\psi^{(1)}(a+1) + 2\psi^{(0)}(a+1)\psi^{(1)}(a+1) \right] + \cdots . \quad (13)$$

However, there is no such identity, and, as it stands (ignoring the enigmatic dots at the end), it is false.

In Section 4, Gemini employed the Lerch phi function, and then finally declared the desired result. Again, while Gemini showed some promise in its approach, in the end its output was deeply flawed.

The author then tried a more advanced version of this software, namely Google Gemini 3 Pro, available from the website https://use.ai on 23 February 2026 (this required a paid subscription). This software quickly solved the problem, in a sense, by citing a "general identity for harmonic sums":

$$\sum_{k=1}^{\infty} \frac{H_k}{(k+a)^2} = (\gamma + \psi(a))\psi^{(1)}(a) - \frac{1}{2}\psi^{(2)}(a). \quad (14)$$

As it turns out, this identity is correct! But Gemini 3 Pro did not provide any proof or cite any reference. When the present author asked it to provide a reference, it provided only a link to a software tool.

The author then tried Gemini 3 Pro on Problem 1 above, namely to prove that for t not an integer, and integer $p \geq 2$, that

$$\sum_{k=1}^{\infty} \frac{H(k)}{(k+t)^p} = \frac{1}{2(p-1)!} \left(t^{-p-1}p! + (-1)^p 2(\gamma + \psi(0,t))\psi(p-1,t) \right.$$

$$\left. + (-1)^p \sum_{k=1}^{p-2} \binom{p-1}{k} \psi(k,t)\psi(p-1-k,t) - (-1)^p \psi(p,1+t) \right). \quad (15)$$

The actual input was:

```
Given a real number t, please prove that the sum of the harmonic function of k divided by (k+t)^p
equals 1/(p-1)! * (p!/t^(p+1) + (-1)^p * 2 * (gamma + polygamma(0,t))*polygamma(p-1,t)
- (-1)^p * polygamma(p,1+t) + (-1)^p * sum (binomial(p-1,k) * polygamma(k,t) * polygamma(p-1-k,t)),
for k = 1 to p-2.
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Gemini 3 Pro clearly understood the problem, and, after two minutes of thinking, produced some promising-looking output.

In its Step 1, Gemini 3 Pro stated that the proof relies on a fundamental relationship between the harmonic numbers and the digamma function (which is true). It then presented a "generating function" for the sum of harmonic numbers with a linear denominator:

$$f(t) = \sum_{k=1}^{\infty} \frac{H_k}{k+t} = \frac{1}{2} \left[(\gamma + \psi((t))^2 + \psi^{(1)}(t) \right] - \frac{1}{2}\zeta(2). \quad (16)$$

But clearly this cannot be correct, because the summation does not converge. Gemini 3 Pro then continued a derivation, finally producing, for $S_p(t)$ (its notation for the original sum),

$$S_p(t) = \frac{(-1)^p}{(p-1)!}\left[-(\gamma + \psi(t))\psi^{(p-1)}(t) - \frac{1}{2}\psi^{(p)}(t) - \frac{1}{2}\sum_{k=1}^{p-2}\binom{p-1}{j}\psi^{(k)}(t)\psi^{(p-1-k)}(t)\right]. \qquad (17)$$

It then applied the recurrence $\psi^{(p)}(t) = \psi^{(p)}(t+1) + (-1)^{p+1}p!/t^{p+1}$ to produce the result

$$S_p(t) = \frac{1}{2(p-1)!}\left[\frac{p!}{t^{p+1}} + (-1)^p 2(\gamma + \psi(t))\psi^{(p-1)}(t) - (-1)^p\psi^{(p)}(t+1) + (-1)^p\sum_{k=1}^{p-2}\binom{p-1}{k}\psi^{(k)}(t)\psi^{(p-1-k)}(t)\right]. \qquad (18)$$

Interestingly, (18) is correct. It can be easily seen to be equivalent to (15). However, (18) does not follow from (17), because (17) is false: for $t = 1/2$ and $p = 3$, it produces a value over 94 times too high.

So while Gemini 3 Pro's output followed an interesting line of analysis, in the end it also employed faulty reasoning and thus was not reliable.

6 Anthropic Claude

The author then tried the Anthropic Claude (Opus 4.5 version) system, available 25 February 2026 from https://use.ai (this required a paid subscription). The problem posed to Claude was exactly the same as was posed to ChatGPT.

Analysis: Claude correctly transcribed the problem, and then pursued a line of reasoning taken by several of the packages, namely to apply the identity $H_k = \int_0^1 (1 - t^k)/(1 - t)\, dt$. With a brief mention of Fubini's theorem, it converted the original problem to:

$$\sum_{k=1}^{\infty}\frac{H_k}{(5k+1)^2} = \int_0^1 \frac{1}{1-t}\left[\sum_{k=1}^{\infty}\frac{1}{(5k+1)^2} - \sum_{k=1}^{\infty}\frac{t^k}{(5k+1)^2}\right]dt. \qquad (19)$$

Claude evaluated the first sum as $1/25 \cdot \psi^{(1)}(\frac{6}{5})$. For the second sum, Claude gave the result

$$\sum_{k=1}^{\infty}\frac{t^k}{(5k+1)^2} = \frac{t}{25}\Phi\left(t, 2, \frac{6}{5}\right), \qquad (20)$$

where $\Phi(z, s, a) = \sum_{n=0}^{\infty} z^n/(n+a)^s$ is the Lerch phi function. The integral then became

$$\frac{1}{25}\int_0^1 \frac{\psi^{(1)}(\frac{6}{5}) - t\Phi(t, 2, \frac{6}{5})}{1-t}\, dt. \qquad (21)$$

Here Claude expanded the numerator using the enigmatic relation

$$\Phi(t, 2, a) = \psi^{(1)}(a) + (t-1)\left[\gamma\psi^{(1)}(a) + \psi^{(0)}(a) - \frac{1}{2}\psi^{(2)}(a)\right] + O((t-1)^2), \qquad (22)$$

and then claimed the result

$$\sum_{k=1}^{\infty}\frac{H(k)}{(5k+1)^2} = \frac{1}{50}\left(250 + 2\gamma\psi(1, 1/5) + 2\psi(0, 1/5)\psi(1, 1/5) - \psi(2, 6/5)\right). \qquad (23)$$

While this overall line of reasoning seemed promising, the present author was quite disappointed at the lack of details in the evaluation of the integral (21) and formula (22). Lacking a detailed explanation of these key steps, this material is not very useful.

7 Conclusion

In general, the author found that the freely available software models performed rather poorly. More advanced models available via paid subscription did significantly better, demonstrating remarkable skill in analyzing and executing mathematical reasoning. They are greatly improved from just a year or two ago. But even these more advanced models do not yet appear ready for "prime time" as mathematical research assistants *for the type of problem addressed in this study*. Difficulties include:

1. Errors of algebra.

2. Reliance on divergent sums, such as $\sum_{k=1}^{\infty} 1/(k+1)$ and $\sum_{k=1}^{\infty} H(k)/(k+1)$.

3. Reliance on formulas or other results without specific literature citation.

4. Reliance on false formulas and results.

5. Lack of details at key steps in the proof.

6. Lack of numerical double-checks, either for intermediate or final results.

With regards to the last item, while there were one or two instances of numerical checks in the output of the software packages studied here, this could be greatly expanded, which by itself should significantly improve the reliability of these systems on real mathematical problems.

The present author would be the first to acknowledge that this exercise has significant limitations, notably that the models were tested only on a very specific set of problems, and thus these results should not be deemed indicative of the likely success on completely different types of problems. Hopefully, once more researchers document their experiences in this manner, a clearer picture will emerge.

Further, in each case the tested software was a specific version available from the listed site on a specific date. But it is well known that these software packages are rapidly being upgraded, and so almost certainly these results, even with the same models, will be quite different in just a few months from now. We eagerly await these improved versions.

However, one conclusion is clear: For the foreseeable future it will be essential for mathematical users of these tools to keep firmly in mind that they do make mistakes, and that ultimately human users are responsible to ensure that published material (even informally published material on preprint servers) is free from difficulties that could potentially corrupt the mathematical research enterprise.

Either way, we are entering a new era. It is an exciting time for the field!

References

[1] Mohammed Abouzaid, Andrew J. Blumberg, Martin Hairer, Joe Kileel, Tamara G. Kolda, Paul D. Nelson, Daniel Spielman, Nikhil Srivastava, Rachel Ward, Shmuel Weinberger and Lauren Williams, "First Proof," 6 Feb 2026, https://arxiv.org/abs/2602.05192.

[2] David H. Bailey, "Can ChatGPT prove math theorems?," *Math Scholar*, 2 Feb 2023, https://mathscholar.org/2023/02/can-chatgpt-prove-math-theorems/.

[3] David H. Bailey, "DeepSeek: A breakthrough in AI for math (and everything else)," 1 Feb 2025, https://mathscholar.org/2025/02/deepseek-a-breakthrough-in-ai-for-math-and-everything-else/.

[4] David H. Bailey, Ross McPhedran and Bruno Salvy, "General formulas for a class of Euler sums," manuscript, 13 Mar 2026.

[5] Lyndie Chiou, "At secret math meeting, researchers struggle to outsmart AI," *Scientific American*, 6 Jun 2025, https://www.scientificamerican.com/article/inside-the-secret-meeting-where-mathematicians-struggled-to-outsmart-ai/.

[6] Christoph Drosser, "AI will become mathematicians' co-pilot (an interview with Terence Tao)," *Scientific American*, 8 Jun 2024, https://www.scientificamerican.com/article/ai-will-become-mathematicians-co-pilot/.

[7] Tamara Kolda, "The future of math research inn the age of AI," *Silicon Reckonner*, 21 Feb 2026, https://siliconreckoner.substack.com/p/the-future-of-math-research-in-the.

[8] Matteo Wong, "The edge of mathematics," *The Atlantic*, 24 Feb 2026, https://www.theatlantic.com/technology/2026/02/ai-math-terrance-tao/686107/.