# General formulas for a large class of Euler sums

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**Abstract**

Let $H(k) = 1 + 1/2 + 1/3 + \cdots + 1/k$ denote the harmonic function. We conjecture a general formula giving closed-form evaluations, via the polygamma function, for Euler sums of the form

$$\sum_{k=1}^{\infty} \frac{H(k)}{(mk+n)^p}$$

(for integers $m, n \geq 1, \gcd(m,n) = 1$ and $p \geq 2$). We further conjecture a formula for mixed-denominator Euler sums such as (for $m_j, n_j, p_j \geq 1, \gcd(m_j, n_j) = 1, (m_i, n_i) \neq (m_j, n_j), r \geq 2$)

$$\sum_{k=1}^{\infty} \frac{H(k)}{(m_1 k + n_1)^{p_1} (m_2 k + n_2)^{p_2} \cdots (m_r k + n_r)^{p_r}}.$$

We then present variations of these formulas to handle polynomials with irrational and complex roots as well. We do not yet have proofs for these conjectures, but we have confirmed them in hundreds of cases with 300-digit computations. We include computational techniques we have employed for directly computing Euler sums such as the above to high precision, and also present some known techniques for computing Euler's constant, the polygamma function and other underlying functions. We conclude with a catalog of several hundred formulas found by these techniques.

## 1 Introduction

An *Euler sum* is an infinite series involving the harmonic function $H(k) = 1 + 1/2 + 1/3 + \cdots + 1/k$. Such sums arise in mathematical physics, in the study of the Riemann hypothesis and in numerous other contexts. They have been studied since the time of Euler, and more recently in [4, 8, 10, 17, 18]. One notable feature of these sums is that many have surprisingly elegant evaluations, for example:

$$\begin{aligned}
\sum_{k=1}^{\infty} \frac{H(k)}{k^3} &= \frac{5}{4}\zeta(4) = \frac{1}{72}\pi^4 \\
\sum_{k=1}^{\infty} \frac{H(k)}{(k+1)^5} &= \frac{1}{4}\left(3\zeta(6) - 2\zeta(3)^2\right) = \frac{1}{1260}\left(\pi^6 - 630\zeta(3)^2\right) \\
\sum_{k=1}^{\infty} \frac{H(k)^3}{k^4} &= \frac{693}{48}\zeta(7) + 2\zeta(5)\zeta(2) - \frac{51}{4}\zeta(4)\zeta(3) \\
\sum_{k=1}^{\infty} \frac{H(k)}{(k+1)^7} &= \frac{1}{4}\left(5\zeta(8) - 4\zeta(3)\zeta(5)\right) = \frac{1}{4}\left(\frac{\pi^8}{1890} - 4\zeta(3)\zeta(5)\right).
\end{aligned} \tag{1}$$

In an earlier study [7], we presented results for mixed-denominator Euler sums of the form

$$M(m, n_0, n_1, n_2, \ldots, n_t) = \sum_{k=1}^{\infty} \frac{H(k)^m}{k^{n_0}(k+1)^{n_1}(k+2)^{n_2}\cdots(k+t)^{n_t}}, \tag{2}$$

for nonnegative integers $m$ and $(n_i)$, with $m \geq 1$ and $n_0 + n_1 + \cdots + n_t \geq 2$. We showed that these have closed-form evaluations in terms of the Riemann zeta function, at least when $m + n_0 + \cdots + n_t \leq 12$. We also presented a catalog of several hundred formulas, produced using high-precision numerical evaluations of the sums, combined with an integer relation algorithm to find the coefficients.

## 2 Four conjectures

Results such as the above, however promising, leave open the much vaster set of Euler sums with more complicated polynomial denominators. To that end, we present the following conjectured results, which we have found experimentally using a combination of *Wolfram Mathematica*, the Online Encyclopedia of Integer Sequences [16] and high-precision numerical computing, pending further analysis and proof.

*Conjecture 1:* Let $\psi(q, z) = D^{q+1}(\log\Gamma(z))$ denote the polygamma function (here $D^{q+1}$ is the $(q+1)$-th derivative), and let $Q(k, p) = 1/2$ if $p$ is odd and $k = (p-1)/2$, and 1 otherwise. Then for integers $m, n \geq 1, \gcd(m, n) = 1$ and $p \geq 2$,

$$\begin{aligned}\sum_{k=1}^{\infty} \frac{H(k)}{(mk+n)^p} = \frac{1}{2m^p(p-1)!} \Bigg( & (m/n)^{p+1}\, p! + (-1)^p 2(\gamma + \psi(0, n/m))\psi(p-1, n/m) \\ & + (-1)^p 2 \sum_{k=1}^{\lfloor (p-1)/2 \rfloor} Q(k, p) \binom{p-1}{k} \psi(k, n/m)\psi(p-1-k, n/m) - (-1)^p \psi(p, 1+n/m) \Bigg). \end{aligned} \tag{3}$$

We do not yet have a proof for this formula, but it has been numerically verified to nearly 300-digit precision in hundreds of specific cases — see Sections 3, 4 and the Appendix (Section 6, page 12).

*Illustration of Conjecture 1.* Conjecture 1 immediately gives evaluations such as these:

$$\begin{aligned}\sum_{k=1}^{\infty} \frac{H(k)}{(3k+2)^3} &= \frac{1}{864}\left(243 - 16\psi(1, 2/3)^2 - 16\gamma\psi(2, 2/3) - 16\psi(0, 2/3)\psi(2, 2/3) + 8\psi(3, 5/3)\right) \\ \sum_{k=1}^{\infty} \frac{H(k)}{(5k+1)^3} &= \frac{1}{500}\left(3750 - 2\psi(1, 1/5)^2 - 2\gamma\psi(2, 1/5) - 2\psi(0, 1/5)\psi(2, 1/5) + \psi(3, 6/5)\right). \end{aligned} \tag{4}$$

*Conjecture 2:* Any mixed-denominator Euler sum of the form

$$G(m_1, n_1, p_1, \cdots, m_r, n_r, p_r) = \sum_{k=1}^{\infty} \frac{H(k)}{(m_1k+n_1)^{p_1}(m_2k+n_2)^{p_2}\cdots(m_rk+n_r)^{p_r}}, \tag{5}$$

for integers $m_j, n_j, p_j \geq 1, \gcd(m_j, n_j) = 1, (m_i, n_i) \neq (m_j, n_j), r \geq 2$, is given by a rational linear sum of terms based on Conjecture 1. In particular, let $S(m, n, p)$ be the expression on the right-hand side of (3) in Conjecture 1 for $p \geq 2$, and, for the case $p = 1$, define

$$S(m, n, 1) = \frac{1}{2m}\left((m/n)^2 - 2\gamma\psi(0, n/m) - \psi(0, n/m)^2 + \psi(1, 1+n/m)\right). \tag{6}$$

Construct the partial fraction decomposition of the summand of (5), without the $H(k)$:

$$\frac{1}{(m_1k+n_1)^{p_1}(m_2k+n_2)^{p_2}\cdots(m_rk+n_r)^{p_r}} = \frac{C_{1,1}}{m_1k+n_1} + \frac{C_{2,1}}{(m_1k+n_1)^2} + \cdots + \frac{C_{p_1,1}}{(m_1k+n_1)^{p_1}} + \cdots + \frac{C_{1,r}}{m_rk+n_r} + \frac{C_{2,r}}{(m_rk+n_r)^2} + \cdots + \frac{C_{p_r,1}}{(m_rk+n_r)^{p_r}}. \quad (7)$$

For each term $C_{i,j}/(m_jk+n_j)^i$, let $S(m_j,n_j,i)$ be the expression given by (3) or (6) for the parameters $m_j, n_j, i$. Then Conjecture 2 asserts that

$$G(m_1,n_1,p_1,\cdots,m_r,n_r,p_r) = \sum_{j=1}^{r}\sum_{i=1}^{p_j} C_{i,j}S(m_j,n_j,i). \quad (8)$$

Again, we do not yet have proof for this assertion, but we have confirmed it numerically to nearly 300-digit precision in hundreds of cases. We should point out here that the process of partial fraction decomposition employed in the above is of course independent of the numerator, whether it be $H(k), H(k)^2, H(2k)$ or some other expression. A catalog of formulas of this type are in the Appendix, starting on page 14.

*Illustration of Conjecture 2.* Consider the Euler sum

$$\sum_{k=1}^{\infty}\frac{H(k)}{(2k+1)^3(3k+1)^2}. \quad (9)$$

We first seek rational coefficients $A, B, C, D, E$ in the partial fraction expansion

$$\frac{1}{(2k+1)^3(3k+1)^2} = \frac{A}{2k+1} + \frac{B}{(2k+1)^2} + \frac{C}{(2k+1)^3} + \frac{D}{3k+1} + \frac{E}{(3k+1)^2}. \quad (10)$$

This is a very well-known process, but for completeness here are the details: After multiplying both sides of (10) by $(2k+1)^3(3k+1)^2$, this becomes

$$\begin{aligned} 1 &= A(2k+1)^2(3k+1)^2 + B(2k+1)(3k+1)^2 + C(3k+1)^2 + D(2k+1)^3(3k+1) + E(2k+1)^3 \\ &= (A+B+C+D+E) + (10A+8B+6C+9D+6E)k + (37A+21B+9C+30D+12E)k^2 \\ &\quad + (60A+18B+44D+8E)k^3 + (36A+24D)k^4. \end{aligned} \quad (11)$$

Viewing (11) as an identity that must be satisfied for all integers $k$, it follows that each coefficient of $k$ on the left-hand side must match the corresponding coefficient of $k$ on the right-hand side. Thus we are led to the system of equations

$$\begin{aligned} 1 &= A+B+C+D+E \\ 0 &= 10A+8B+6C+9D+6E \\ 0 &= 37A+21B+9C+30D+12E \\ 0 &= 60A+18B+44D+8E \\ 0 &= 36A+24D. \end{aligned} \quad (12)$$

This can be solved by well-known linear system methods, yielding the solution $A = 108, B = 24, C = 4, D = -162, E = 27$.

Next, note that formulas (3) and (6) above yield

$$\begin{aligned}
S(2,1,1) &= \frac{1}{4}\left(4 - 2\gamma\psi(0,1/2) - \psi(0,1/2)^2 + \psi(1,3/2)\right)\\
S(2,1,2) &= \frac{1}{8}\left(16 + 2\gamma\psi(1,1/2) + 2\psi(0,1/2)\psi(1,1/2) - \psi(2,3/2)\right)\\
S(2,1,3) &= \frac{1}{32}\left(96 - 2\gamma\psi(2,1/2) - 2\psi(0,1/2)\psi(2,1/2) - 2\psi(1,1/2)^2 + \psi(3,3/2)\right)\\
S(3,1,1) &= \frac{1}{6}\left(9 - 2\gamma\psi(0,1/3) - \psi(0,1/3)^2 + \psi(1,4/3)\right)\\
S(3,1,2) &= \frac{1}{18}\left(54 + 2\gamma\psi(1,1/3) + 2\psi(0,1/3)\psi(1,1/3) - \psi(2,4/3)\right). \qquad (13)
\end{aligned}$$

Thus formula (8) of Conjecture 2 asserts that

$$\begin{aligned}
\sum_{k=1}^{\infty}\frac{H(k)}{(2k+1)^3(3k+1)^2} &= \frac{1}{4}\left(-24 - 216\gamma\psi(0,1/2) - 108\psi(0,1/2)^2 + 108\psi(1,3/2) + 24\gamma\psi(1,1/2)\right.\\
&\quad +24\psi(0,1/2)\psi(1,1/2) - 12\psi(2,3/2) - \gamma\psi(2,1/2) - \psi(0,1/2)\psi(2,1/2)\\
&\quad +\psi(1,1/2)^2 + 216\gamma\psi(0,1/3) + 108\psi(0,1/3)^2 - 108\psi(1,4/3)\\
&\quad \left.+12\gamma\psi(1,1/3) + 12\psi(0,1/3)\psi(1,1/3) - 6\psi(2,4/3)\right)\\
&= 0.0026439694745412157544 4\ldots, \qquad (14)
\end{aligned}$$

a value that we have found to agree to more than 280 digits with a direct computation of the Euler sum. Many more such evaluations are given below in the Appendix (Section 6).

We note in passing that Euler sums corresponding to the terms $A/(2k+1)$ and $D/(3k+1)$ in (10), namely $\sum_{k=1}^{\infty} H(k)/(2k+1)$ and $\sum_{k=1}^{\infty} H(k)/(3k+1)$, do not converge. But since $A = 108$ and $D = -162$, respectively, their sum becomes

$$\sum_{k=1}^{\infty} H(k)\left(\frac{108}{2k+1} - \frac{162}{3k+1}\right) = -\sum_{k=1}^{\infty}\frac{H(k)}{6k^2+5k+1}, \qquad (15)$$

which does converge, as it must, since the left-hand term of (10) and all other right-hand terms, if written as individual Euler sums, do converge.

It is clear that these formulas have broader applicability. In particular, we conjecture the following:

*Conjecture 3.* For nonzero $t$ (real or complex) not a negative integer, integer $p \geq 2$ and $Q(k,p)$ as in Conjecture 1,

$$\begin{aligned}
\sum_{k=1}^{\infty}\frac{H(k)}{(k+t)^p} &= \frac{1}{2(p-1)!}\left((1/t)^{p+1}\,p! + (-1)^p 2(\gamma + \psi(0,t))\psi(p-1,t)\right.\\
&\quad \left.+(-1)^p 2\sum_{k=1}^{\lfloor (p-1)/2\rfloor} Q(k,p)\binom{p-1}{k}\psi(k,t)\psi(p-1-k,t) - (-1)^p\psi(p,1+t)\right). \qquad (16)
\end{aligned}$$

*Conjecture 4.* In analogy to Conjecture 2, consider the mixed-denominator Euler sum

$$K(t_1,p_1,\cdots,t_r,p_r) = \sum_{k=1}^{\infty}\frac{H(k)}{(k+t_1)^{p_1}(k+t_2)^{p_2}\cdots(k+t_r)^{p_r}}, \qquad (17)$$

for nonzero $t_j$ (real or complex) not negative integers, $t_i \neq t_j$, positive integers $p_j$ and $r \geq 2$. Let $T(t,p)$ be the expression given by (16) for $p \geq 2$ and, for $p = 1$, set

$$T(t,1) = \frac{1}{2}\left((1/t)^2 - 2\gamma\psi(0,t) - \psi(0,t)^2 + \psi(1,1+t)\right). \qquad (18)$$

Construct the partial fraction decomposition of the summand of (5), without the $H(k)$:

$$\frac{1}{(k+t_1)^{p_1}(k+t_2)^{p_2}\cdots(k+t_r)^{p_r}} = \frac{C_{1,1}}{k+t_1} + \frac{C_{2,1}}{(k+t_1)^2} + \cdots + \frac{C_{p_1,1}}{(k+t_1)^{p_1}} + \cdots$$
$$+ \frac{C_{1,r}}{k+t_r} + \frac{C_{2,r}}{(k+t_r)^2} + \cdots + \frac{C_{p_r,1}}{(k+t_r)^{p_r}}. \tag{19}$$

For each term $C_{i,j}/(k+t_j)^i$, let $T(t_j,i)$ be the expression given by (16) or (18) for the parameters $t_j, i$. Then Conjecture 4 asserts that

$$K(t_1,p_1,\cdots,t_r,p_r) = \sum_{j=1}^{r}\sum_{i=1}^{p_j} C_{i,j}T(t_j,i). \tag{20}$$

In short, Conjectures 3 and 4 extend the results implied by Conjectures 1 and 2 to the case of denominator polynomials with irrational and complex roots. As yet we have no proof for these assertions, but they have been affirmed, via high-precision numerical computation, in numerous specific cases. A brief catalog of formulas of this type are in the Appendix, starting on page 48.

*Illustration of Conjectures 3 and 4.* Consider the Euler sum

$$\sum_{k=1}^{\infty}\frac{H(k)}{k^2+3k+1}. \tag{21}$$

Note that $k^2+3k+1 = (k+(3-\sqrt{5})/2)(k+(3+\sqrt{5})/2)$. A partial fraction decomposition yields

$$\frac{1}{k^2+3k+1} = \frac{1}{\sqrt{5}}\left(\frac{1}{k+(3-\sqrt{5})/2)} - \frac{1}{k+(3+\sqrt{5})/2}\right). \tag{22}$$

According to formula (18) of Conjecture 4,

$$T((3-\sqrt{5})/2),1) = \frac{4}{(3-\sqrt{5})^2} - \gamma\psi(0,(3-\sqrt{5})/2) - \frac{1}{2}\psi(0,(3-\sqrt{5})/2)^2 + \frac{1}{2}\psi(1,1+(3-\sqrt{5})$$
$$T((3+\sqrt{5})/2),1) = \frac{4}{(3+\sqrt{5})^2} - \gamma\psi(0,(3+\sqrt{5})/2) - \frac{1}{2}\psi(0,(3+\sqrt{5})/2)^2 + \frac{1}{2}\psi(1,1+(3+\sqrt{5}). \tag{23}$$

Thus by formula (20),

$$\sum_{k=1}^{\infty}\frac{H(k)}{k^2+3k+1} = \frac{1}{\sqrt{5}}\Big(T((3-\sqrt{5})/2),1) - T((3+\sqrt{5})/2),1)\Big)$$
$$= \frac{1}{2\sqrt{5}}\Big(3\sqrt{5} - 2\gamma\psi(0,(3-\sqrt{5})/2) - \psi(0,(3-\sqrt{5})/2)^2 + \psi(1,(5-\sqrt{5})/2)$$
$$+2\gamma\psi(0,(3+\sqrt{5})/2) + \psi(0,(3+\sqrt{5}))^2 - \psi(1,(5+\sqrt{5})/2)\Big). \tag{24}$$

As a somewhat more challenging example, consider

$$\sum_{k=1}^{\infty}\frac{H(k)}{(k^2+3k+1)^2}. \tag{25}$$

Note that $(k^2+3k+1)^2 = (k+(3-\sqrt{5})/2)^2(k+(3+\sqrt{5})/2)^2$. A partial fraction decomposition yields

$$\frac{1}{(k^2+3k+1)^2} = \frac{-2}{5\sqrt{5}(x+(3-\sqrt{5})/2)} + \frac{1}{5(x+(3-\sqrt{5})/2)^2} + \frac{2}{5\sqrt{5}(x+(3+\sqrt{5})/2)} + \frac{1}{5(x+(3+\sqrt{5})/2)^2}. \tag{26}$$

Let $q_d = \sqrt{5},\ q_1 = (3-\sqrt{5})/2,\ q_2 = (3+\sqrt{5})/2$. Then, according to (16), (18) and (20),

$$\sum_{k=1}^{\infty} \frac{H(k)}{(k^2+3k+1)^2} = \frac{-1}{10}\left(400/(q_1^2 q_d) - 4\gamma\psi(0,q_1)/q_d - 2\psi(0,q_1)^2/q_d + 2\psi(1,1+q_1)/q_d - 70/q_1^3\right.$$
$$-2\gamma\psi(1,q_1) - 2\psi(0,q_1)\psi(1,q_1) + \psi(2,1+q_1) + 4\gamma\psi(0,q_2)/q_d + 2\psi(0,q_2)^2/q_d$$
$$\left.-2\psi(1,1+q_2)/q_d - 2\gamma\psi(1,q_2) - 2\psi(0,q_2)\psi(1,q_2) + \psi(2,1+q_2)\right). \tag{27}$$

Finally, consider this example, which has complex roots:

$$\sum_{k=1}^{\infty} \frac{H(k)}{k^3+1}. \tag{28}$$

Let $(-1, c_1 = (1+i\sqrt{3})/2, c_2 = (1-i\sqrt{3})/2)$ be the cubic roots of -1, so that $(k+1)(k-c_1)(k-c_2) = k^3+1$. A partial fraction decomposition yields

$$\frac{1}{k^3+1} = \frac{1}{3(k+1)} - \frac{c_1}{3(k-c_1)} - \frac{c_2}{3(k-c_2)}. \tag{29}$$

Recall that $\psi(0,1) = \gamma$ and $\psi(1,2) = \pi^2/6 - 1$. Then

$$\sum_{k=1}^{\infty} \frac{H(k)}{k^3+1} = \frac{1}{3}T(1,1) - \frac{c_1}{3}T(-c_1,1) - \frac{c_2}{3}T(-c_2,1)$$
$$= \frac{1}{6}\left(-c_1 + c_2 + \gamma^2 + \pi^2/6 + 2c_1\gamma\psi(0,-c_1) + c_1\psi(0,-c_1)^2 + 2c_2\gamma\psi(0,-c_2)\right.$$
$$\left.+c_2\psi(0,-c_2)^2 - c_1\psi(1,1-c_1) - c_2\psi(1,1-c_2)\right)$$
$$= 0.828902143400992508742\ldots. \tag{30}$$

## 3 Directly computing mixed-denominator Euler sums

The research leading to the conjectures in the previous section relied on direct high-precision numerical evaluations of the Euler sums, combined with an integer relation computation to produce the coefficients of conjectured right-hand side terms. Also, even with the conjectures in hand, we have found that it is often easier to use numerical methods to obtain the formulas. In fact, the catalog of formulas in the Appendix was obtained by this process, after checking in each case that the numerical value matches the result given by the conjectures. In this section, we describe the computational scheme we have employed for these evaluations. It is similar to but somewhat different than the scheme employed in [7].

The Euler-Maclaurin summation formula [1, pg. 285] approximates a summation with an integral and high-order corrections (here $f(t)$ is assumed to have $(2s+2)$-th order derivatives on $[a,b]$):

$$\sum_{j=a}^{b} f(j) = \int_a^b f(t)\,\mathrm{d}t + \frac{1}{2}\left(f(a)+f(b)\right) + \sum_{j=1}^{s} \frac{B_{2j}\left(D^{2j-1}f(b) - D^{2j-1}f(a)\right)}{(2j)!} + R_s(a,b), \tag{31}$$

where $B_k$ is the $k$-th Bernoulli number [11], $D^k f(a)$ is the $k$-th derivative of $f(t)$ evaluated at $t=a$, and

$$R_s(a,b) = \frac{-1}{(2s+2)!}\int_a^b B_{2s+2}(t-[t])D^{2s+2}f(t)\,\mathrm{d}t, \tag{32}$$

where $[\cdot]$ denotes greatest integer and $B_k(\cdot)$ is the $k$-th Bernoulli polynomial [11] (note $B_k = B_k(0)$).

Applying the Euler-Maclaurin summation formula to the harmonic function $H(t) = \sum_{j=1}^{t} 1/j$ yields

$$H(t) = \gamma + \log(t) + \frac{1}{2t} + \sum_{j=1}^{s} \frac{B_{2j}}{2jt^{2j}} + R_s(t), \tag{33}$$

where $\gamma = 0.5772156649\ldots$ is Euler's constant and $|R_s(t)| \le |B_{2s+2}|/((2s+2)t^{2s+2})$. In the computations for the present study, we set $s = 18$, so that $H(t)$ is approximated by

$$\begin{aligned}\hat{H}(t) = \gamma + \log(t) &+ \frac{1}{2t} - \frac{1}{12t^2} + \frac{1}{120t^4} - \frac{1}{252t^6} + \frac{1}{240t^8} - \frac{1}{132t^{10}} + \frac{691}{32760t^{12}} - \frac{1}{12t^{14}} \\ &+ \frac{3617}{8160t^{16}} - \frac{43867}{14364t^{18}} + \frac{174611}{6600t^{20}} - \frac{77683}{276t^{22}} + \frac{236364091}{65520t^{24}} - \frac{657931}{12t^{26}} + \frac{3392780147}{3480t^{28}} \\ &- \frac{1723168255201}{85932t^{30}} + \frac{7709321041217}{16320t^{32}} - \frac{151628697551}{12t^{34}} + \frac{26315271553053477373}{69090840t^{36}},\end{aligned} \tag{34}$$

which approximates $H(t)$ to within roughly $t^{-38}$. Now given a mixed-denominator Euler sum such as

$$G(m_1, n_1, p_1, m_2, n_2, p_2) = \sum_{k=1}^{\infty} \frac{H(k)}{(m_1 k + n_1)^{p_1}(m_2 k + n_2)^{p_2}}, \tag{35}$$

denote $\hat{G}(t) = \hat{H}(t)/((m_1 k + n_1)^{p_1}(m_2 k + n_2)^{p_2})$. Using the Euler-Maclaurin summation formula (31) once again, we can write

$$\begin{aligned}G(m_1, n_1, p_1, m_2, n_2, p_2) &= \sum_{j=1}^{k} \frac{H(j)}{(m_1 j + n_1)^{p_1}(m_2 j + n_2)^{p_2}} + \sum_{j=k+1}^{\infty} \frac{H(j)}{(m_1 k + n_1)^{p_1}(m_2 k + n_2)^{p_2}} \\ &\approx \sum_{j=1}^{k} \frac{H(j)}{(m_1 k + n_1)^{p_1}(m_2 k + n_2)^{p_2}} + \sum_{j=k+1}^{\infty} \hat{G}(j) \\ &\approx \sum_{j=1}^{k} \frac{H(j)}{(m_1 k + n_1)^{p_1}(m_2 k + n_2)^{p_2}} + \int_{k+1}^{\infty} \hat{G}(t)\,\mathrm{d}t + \frac{1}{2}\hat{G}(k+1) - \sum_{j=1}^{s} \frac{B_{2j} D^{2j-1}\hat{G}(k+1)}{(2j)!},\end{aligned} \tag{36}$$

where $s = 18$, which is accurate to within roughly $k^{-38}$. We set $k = 10^8$, so the approximation in the last line of (36) is correct to within roughly $10^{-294}$.

We evaluated the first three terms of (36) using multiprecision software written by one of the present authors [2], and we evaluated the fourth term using *Wolfram Mathematica*. This process produces a result typically accurate to roughly 280 digits. With the high-precision numerical value in hand, the multipair PSLQ integer relation algorithm was employed to find the coefficients for the resulting formula. For additional details, see [3, 5, 7].

# 4 Computing Euler's constant and the polygamma function

The formulas given by Conjectures 1, 2, 3 and 4 involve both Euler's constant and the polygamma function, which is defined as $\psi(q, z) = D^{q+1}(\log \Gamma(z))$, where $\Gamma(z)$ denotes the gamma function. The special case $\psi(0, z)$ is also known as the digamma function. *Maple* and *Mathematica* can evaluate these to arbitrary precision, but not all researchers have access to this commercial software, and others may wish to explore these relations with custom code as we have done.

To that end, we present here a brief summary of techniques for computing high-precision values of Euler's constant, digamma, polygamma, together with several other underlying functions, focusing on practical techniques for research use. These algorithms are for real arguments, but in most cases can be extended to complex arguments with minor changes. We only assume some arbitrary-precision software with exp, sin, log, etc. (for example, the MPFR package [13]). For additional details, see [2].

*Euler's constant.* Euler's constant $\gamma = 0.5772156649\ldots$ can be calculated with the following formula [2].

For $B$ bits of precision, select the integer $N \geq \log_2(B \log 2)$. Then

$$\gamma \approx \frac{2^N}{e^{2^N}} \sum_{m=0}^{\infty} \frac{2^{mN}}{(m+1)!} \sum_{t=0}^{m} \frac{1}{t+1} \; - \; N \log 2. \tag{37}$$

*Digamma.* The digamma function can be computed directly from its definition as

$$\psi(0,x) = \Gamma'(x)/\Gamma(x) \approx \frac{\Gamma(x+h) - \Gamma(x)}{h\Gamma(x)}, \tag{38}$$

where $h = 2^{-P}$ ($P$ is the working precision in bits), and the operation is performed using $2P$-bit precision (see *Gamma function* below).

A significantly faster scheme for digamma is to use the following asymptotic formula [11, 5.11.2]:

$$\psi(0,x) \approx \log(x) - \frac{1}{2x} - \sum_{k=1}^{\infty} \frac{B_{2k}}{2kx^{2k}}, \tag{39}$$

where $B_{2k}$ are the even Bernoulli numbers (see *Even Bernoulli numbers* below). For this to be practical, however, the input argument must first be shifted up using the recursion $\psi(0,x+1) = \psi(0,x) + 1/x$. One choice is to repeat this recursion $0.45P$ times, where $P$ is the working precision in bits.

*Polygamma.* To compute the polygamma function $\psi(q,z)$ for integer $q \geq 1$, first reduce the second argument to the range $(0,1]$ by applying the recurrence

$$\psi(q,z) = \psi(q,z+1) - (-1)^q q! z^{-q-1}. \tag{40}$$

Then the polygamma function can be evaluated from the Hurwitz zeta function (see *Hurwitz zeta function* below) via the formula [14]

$$\psi(q,z) = (-1)^{q+1} q! \zeta(q+1,z). \tag{41}$$

*Gamma function.* If the input $t$ is a positive integer, then $\Gamma(t) = (t-1)!$. If not, use the recursion $\Gamma(t+1) = t\Gamma(t)$ to reduce the argument (positive or negative) to the interval $(0,1)$. If $t$ is very close to 1, then employ this formula [11, 5.7.3]:

$$\log(\Gamma(1+x)) = -\log(1+x) + x(1-\gamma) + \sum_{k=2}^{\infty} (-1)^k (\zeta(k) - 1) \frac{x^k}{k}, \tag{42}$$

where $x = t-1$ and $\gamma$ is Euler's constant. For the zeta function, see *Zeta function* below. For other values of $t$, employ this scheme due to Potter [15]: Define $\alpha = 2\,\mathrm{int}\,(b/4 \cdot \log 2 + 1)$, where $b$ is the number of bits of precision and int is greatest integer. Define the Pochhammer symbol as $(t)_k = t(t+1)(t+2)\cdots(t+k-1)$. Then define

$$\begin{aligned} A(t,\alpha) &= \left(\frac{\alpha}{2}\right)^t t \sum_{k=0}^{\infty} \frac{(\alpha^2/4)^k}{k!(t)_{k+1}} \\ B(t,\alpha) &= \left(\frac{\alpha}{2}\right)^{-t} (-t) \sum_{k=0}^{\infty} \frac{(\alpha^2/4)^k}{k!(-t)_{k+1}}. \end{aligned} \tag{43}$$

With these definitions, the gamma function can then be computed as

$$\Gamma(t) = \sqrt{\frac{\pi A(t,\alpha)}{t \sin(\pi t) B(t,\alpha)}}. \tag{44}$$

*Hurwitz zeta function.* The Hurwitz zeta function is defined as $\zeta(s,a) = \sum_{k=0}^{\infty} 1/(k+a)^s$. If $a = m$ is a positive integer, then $\zeta(s,m) = \zeta(m) - \sum_{k=1}^{m-1} 1/k^s$ (see *Zeta function* below). If $a+m$ is some other positive real, for $0 < a < 1$, then $\zeta(s,a+m) = \zeta(s,a) - \sum_{k=0}^{m-1} 1/(k+a)^s$. Thus we can assume that $0 < a < 1$. Then one can employ the following, modified from Crandall [6]: Let $\sigma(x)$ denote $\operatorname{sign}(x)$. Then for $s \neq 1, 0, -1, -2, \cdots$,

$$\begin{aligned}\zeta(s,a) = &\frac{\sqrt{\pi}\,\pi^{(s-1)/2}}{(s-1)\,\Gamma(s/2)} \\ &+ \frac{1}{2}\sum_{n=-\infty}^{\infty}\left(\frac{\Gamma\left(\frac{s}{2},\pi\,(n+a)^2\right)}{\Gamma(s/2)} + \sigma(n+a)\frac{\Gamma\left(\frac{s+1}{2},\pi\,(n+a)^2\right)}{\Gamma((s+1)/2)}\right)\frac{1}{|n+a|^s} \\ &+ \pi^{s-1/2}\sum_{m=1}^{\infty}\frac{1}{m^{1-s}}\left(\frac{\Gamma\left(\frac{1-s}{2},\frac{m^2\pi^2}{\pi}\right)}{\Gamma(s/2)}\cos(2\,\pi m a) + \frac{\Gamma\left(1-\frac{s}{2},\frac{m^2\pi^2}{\pi}\right)}{\Gamma((s+1)/2)}\sin(2\,\pi\,m a)\right). \end{aligned} \tag{45}$$

Although each of the individual terms is relatively expensive to compute due to the incomplete gamma functions (see *Incomplete gamma function* below), convergence is quite rapid.

An even faster formula for the Hurwitz zeta function, which relies on precomputed even Bernoulli numbers (see *Even Bernoulli numbers* below), is as follows. Select an integer $q > 0$ (one choice is $q = \operatorname{nint}(0.6d)$, where $d$ is the precision in digits and nint is nearest integer). Then

$$\begin{aligned}\zeta(s,a) = &\sum_{k=0}^{q-1}\frac{1}{(a+k)^s} + \frac{1}{(s-1)(a+q)^{s-1}} + \frac{1}{2(a+q)^s} \\ &+ \sum_{k=1}^{\infty}\frac{B_{2k}\,s(s+1)\cdots(s+2k-2)}{(2k)!(a+q)^{s+2k-1}}. \end{aligned} \tag{46}$$

*Incomplete gamma function.* The incomplete gamma function can be computed as follows: For modest-sized positive arguments, use this formula [11, 8.7.3]:

$$\Gamma(a,z) = \Gamma(a)\left(1 - \frac{z^a}{e^z}\sum_{k=0}^{\infty}\frac{z^k}{\Gamma(a+k+1)}\right). \tag{47}$$

Note, that although formula (47) involves the gamma function (see *Gamma function above*), this can be called once to compute $\Gamma(a)$, after which the recursion $\Gamma(t+1) = t\Gamma(t)$ can be applied for all other terms. For large values of $z$, use this asymptotic formula [11, 8.11.2]:

$$\Gamma(a,z) \approx \frac{z^{a-1}}{e^z}\sum_{k=0}^{\infty}\frac{(-1)^k(1-a)_k}{z^k}. \tag{48}$$

For the case $a = 0$, use this formula:

$$\Gamma(0,z) = -\gamma - \log(|z|) + \exp(-z/2)\sum_{k=1}^{\infty}\frac{z^k}{2^{k-1}k!}\sum_{j=0}^{\lfloor(k-1)/2\rfloor}\frac{1}{2j+1}. \tag{49}$$

For $a = -1, -2, \cdots$, use this formula [12]:

$$\Gamma(-n,z) = \frac{1}{n!}\left((-1)^n\Gamma(0,z) + \frac{1}{e^z z^n}\sum_{k=0}^{n-1}(-1)^k(n-k-1)!z^k\right). \tag{50}$$

One difficulty is that cancellation may occur in formulas (49) and (50), so for certain arguments the working precision must be increased, possibly to as much as 3X the requested permission.

*Zeta function.* For large positive arguments $s$ (one choice is the condition $s > B\log(2)/\log(2B/3)$), where $B$ is the precision in digits), it suffices to use the definition of zeta, namely

$$\zeta(s) = \sum_{n=1}^{\infty} \frac{1}{n^s}. \tag{51}$$

For modest-sized arguments, the zeta function can be evaluated by means of this formula, due to Peter Borwein [9]: Select $n$ to be the number of digits of precision required for the result. Define

$$e_j = (-1)^j \left( \sum_{k=0}^{j-n} \frac{n!}{k!(n-k)!} - 2^n \right), \tag{52}$$

where the empty sum is zero. Then

$$\zeta(s) \approx \frac{-1}{2^n(1-2^{1-s})} \sum_{j=0}^{2n-1} \frac{e_j}{(j+1)^s}. \tag{53}$$

There is some loss of precision due to cancelation with these formulas, so a somewhat higher working precision is required.

An even faster algorithm for the zeta function, based on the Euler-Maclaurin summation formula, can be derived from the following [11, 25.2.9]: Select an integer parameter $N > 0$ (one choice is $N = \operatorname{nint}(0.6d)$, where $d$ is the precision in digits and nint is nearest integer). Then

$$\zeta(s) = \sum_{k=1}^{N} \frac{1}{k^s} + \frac{1}{(s-1)N^{s-1}} - \frac{1}{2N^s} + \sum_{k=1}^{\infty} \binom{s+2k-2}{2k-1} \frac{B_{2k}}{2kN^{s-1+2k}}, \tag{54}$$

where $B_{2k}$ are the even Bernoulli numbers (see *Even Bernoulli numbers* below).

*Even Bernoulli numbers.* The first $n$ even Bernoulli numbers can be computed as follows [2]: First, compute the zetas $\{\zeta(2k),\ 0 \le k \le n\}$ based on the following well-known formulas:

$$\begin{aligned}
\coth(\pi x) &= \cosh(\pi x)/\sinh(\pi x) \\
&= \frac{1}{\pi x} \cdot \frac{1 + (\pi x)^2/2! + (\pi x)^4/4! + \cdots}{1 + (\pi x)^2/3! + (\pi x)^4/5! + \cdots} \\
&= \frac{1}{\pi x} \cdot \left(1 + (\pi x)^2/3 - (\pi x)^4/45 + 2(\pi x)^6/945 - \cdots\right) \\
&= \frac{2}{\pi x} \sum_{k=1}^{\infty} (-1)^{k+1} \zeta(2k) x^{2k}.
\end{aligned} \tag{55}$$

The strategy here is to calculate the coefficients of this series by polynomial operations in the second line above. Polynomial division is performed by computing the reciprocal of the denominator polynomial, by a polynomial Newton iteration, as follows. Let $N(x)$ be the polynomial approximation to the numerator series; let $D(x)$ be a polynomial approximation to the numerator numerator series; and let $Q_k(x)$ be polynomial approximations to $R(x) = 1/D(x)$. Then iterate:

$$Q_{k+1} = Q_k(x) + [1 - D(x)Q_k(x)]Q_k(x). \tag{56}$$

The reciprocal polynomial $R(x)$ produced by this process is then multiplied by the numerator polynomial $N(x)$ to yield an approximation to the quotient series. The even zeta values are then the coefficients of this series, scaled according to the formula (55) above. Once the even integer zeta values have been computed, the even Bernoulli coefficients are computed via this well-known formula (for $n > 0$):

$$B_{2n} = \frac{(-1)^{n-1}\ 2(2n)!\zeta(2n)}{(2\pi)^{2n}}. \tag{57}$$

## 5 Conclusion

In this paper, we have presented formulas and computational techniques for evaluating a large class of Euler sums. Although we do not yet have proofs for these formulas, we have confirmed them numerically, to very high precision, in hundreds of specific cases. We should note here that *Wolfram Mathematica* appears to be able to produce many, but by no means all, of these results. However, we have found that *Mathematica* often "simplifies" expressions in ways that make it hard to ascertain general patterns. In short, we have found *Mathematica* very useful in this research, but not sufficient by itself for resolving these questions.

In spite of such results, it is clear there is still much that is not known. For example, one drawback of the above approach is that it relies heavily on partial fraction decompositions. It is not known whether reasonable formulas exist that do not require this step. In any event, there are numerous other classes of Euler sums that are not covered by these results. For example, we are not aware of any results of this nature for relatively simple sums such as

$$\sum_{k=1}^{\infty} \frac{H(k)^2}{(mk+n)^p}, \quad \sum_{k=1}^{\infty} \frac{H(2k)}{(mk+n)^p}, \tag{58}$$

for positive integers $m \geq 3, \gcd(m,n) = 1$ and $p \geq 2$. So there is still much to ponder.

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# 6 Appendix: Catalog of formulas using the Conjectures

We present here a selection of formulas that we have produced according to Conjectures 1, 2, 3 and 4. In each case, we computed the sum numerically in two ways: (a) by the direct scheme described in Section 3, and (b) by applying Conjecture 1, 2, 3 or 4 as appropriate. The multipair PSLQ algorithm, applied to the computed value and the hypothesized right-hand side constants, then generated the relations below.

*To guard against the possibility of transcription errors, the LaTeX code for these formulas was produced by a program directly from the computer output, and is included here without any alteration.*

$$\sum_{k=1}^{\infty} \frac{H(k)}{(2k+1)^2} = \frac{1}{8}\left(16 + 2\gamma\psi(1,1/2) + 2\psi(0,1/2)\psi(1,1/2) - 1\psi(2,3/2)\right) \tag{59}$$

$$\sum_{k=1}^{\infty} \frac{H(k)}{(2k+1)^3} = \frac{1}{16}\left(1\psi(1,1/2)^2 - 1\gamma\psi(2,1/2) - 1\psi(0,1/2)\psi(2,1/2)\right) \tag{60}$$

$$\sum_{k=1}^{\infty} \frac{H(k)}{(3k+1)^2} = \frac{1}{18}\left(54 + 2\gamma\psi(1,1/3) + 2\psi(0,1/3)\psi(1,1/3) - 1\psi(2,4/3)\right) \tag{61}$$

$$\sum_{k=1}^{\infty} \frac{H(k)}{(3k+1)^3} = \frac{1}{108}\left(486 - 2\psi(1,1/3)^2 - 2\gamma\psi(2,1/3) - 2\psi(0,1/3)\psi(2,1/3) + 1\psi(3,4/3)\right) \tag{62}$$

$$\sum_{k=1}^{\infty} \frac{H(k)}{(3k+2)^2} = \frac{1}{72}\left(27 + 8\gamma\psi(1,2/3) + 8\psi(0,2/3)\psi(1,2/3) - 4\psi(2,5/3)\right) \tag{63}$$

$$\sum_{k=1}^{\infty} \frac{H(k)}{(3k+2)^3} = \frac{1}{864}\left(243 - 16\psi(1,2/3)^2 - 16\gamma\psi(2,2/3) - 16\psi(0,2/3)\psi(2,2/3) + 8\psi(3,5/3)\right) \tag{64}$$

$$\sum_{k=1}^{\infty} \frac{H(k)}{(4k+1)^2} = \frac{1}{32}\left(128 + 2\gamma\psi(1,1/4) + 2\psi(0,1/4)\psi(1,1/4) - 1\psi(2,5/4)\right) \tag{65}$$

$$\sum_{k=1}^{\infty} \frac{H(k)}{(4k+1)^3} = \frac{1}{256}\left(1536 - 2\psi(1,1/4)^2 - 2\gamma\psi(2,1/4) - 2\psi(0,1/4)\psi(2,1/4) + 1\psi(3,5/4)\right) \tag{66}$$

$$\sum_{k=1}^{\infty} \frac{H(k)}{(4k+3)^2} = \frac{1}{864}\left(128 + 54\gamma\psi(1,3/4) + 54\psi(0,3/4)\psi(1,3/4) - 27\psi(2,7/4)\right) \tag{67}$$

$$\sum_{k=1}^{\infty} \frac{H(k)}{(4k+3)^3} = \frac{1}{6912}\left(512 - 54\psi(1,3/4)^2 - 54\gamma\psi(2,3/4) - 54\psi(0,3/4)\psi(2,3/4) + 27\psi(3,7/4)\right) \tag{68}$$

$$\sum_{k=1}^{\infty} \frac{H(k)}{(5k+1)^2} = \frac{1}{50}\left(250 + 2\gamma\psi(1,1/5) + 2\psi(0,1/5)\psi(1,1/5) - 1\psi(2,6/5)\right) \tag{69}$$

$$\sum_{k=1}^{\infty} \frac{H(k)}{(5k+1)^3} = \frac{1}{500}\left(3750 - 2\psi(1,1/5)^2 - 2\gamma\psi(2,1/5) - 2\psi(0,1/5)\psi(2,1/5) + 1\psi(3,6/5)\right) \tag{70}$$

$$\sum_{k=1}^{\infty} \frac{H(k)}{(5k+2)^2} = \frac{1}{200}\left(125 + 8\gamma\psi(1,2/5) + 8\psi(0,2/5)\psi(1,2/5) - 4\psi(2,7/5)\right) \tag{71}$$

$$\sum_{k=1}^{\infty} \frac{H(k)}{(5k+2)^3} = \frac{1}{4000}\left(1875 - 16\psi(1,2/5)^2 - 16\gamma\psi(2,2/5) - 16\psi(0,2/5)\psi(2,2/5) + 8\psi(3,7/5)\right) \tag{72}$$

$$\sum_{k=1}^{\infty} \frac{H(k)}{(5k+3)^2} = \frac{1}{1350}\left(250 + 54\gamma\psi(1,3/5) + 54\psi(0,3/5)\psi(1,3/5) - 27\psi(2,8/5)\right) \tag{73}$$

$$\sum_{k=1}^{\infty} \frac{H(k)}{(5k+3)^3} = \frac{1}{13500}\left(1250 - 54\psi(1,3/5)^2 - 54\gamma\psi(2,3/5) - 54\psi(0,3/5)\psi(2,3/5) + 27\psi(3,8/5)\right) \tag{74}$$

$$\sum_{k=1}^{\infty} \frac{H(k)}{(5k+4)^2} = \frac{1}{1600}\left(125 + 64\gamma\psi(1,4/5) + 64\psi(0,4/5)\psi(1,4/5) - 32\psi(2,9/5)\right) \tag{75}$$

$$\sum_{k=1}^{\infty} \frac{H(k)}{(5k+4)^3} = \frac{1}{64000}\left(1875 - 256\psi(1,4/5)^2 - 256\gamma\psi(2,4/5) - 256\psi(0,4/5)\psi(2,4/5) + 128\psi(3,9/5)\right) \tag{76}$$

$$\sum_{k=1}^{\infty}\frac{H(k)}{(2k+1)(3k+1)} = \frac{1}{2}\left(5+2\gamma\psi(0,1/2)+1\psi(0,1/2)^2-1\psi(1,3/2)-2\gamma\psi(0,1/3)-1\psi(0,1/3)^2\right.$$
$$\left.+1\psi(1,4/3)\right) \tag{77}$$

$$\sum_{k=1}^{\infty}\frac{H(k)}{(2k+1)(3k+1)^2} = \frac{-1}{6}\left(-24+12\gamma\psi(0,1/2)+6\psi(0,1/2)^2-6\psi(1,3/2)-12\gamma\psi(0,1/3)-6\psi(0,1/3)^2\right.$$
$$\left.+6\psi(1,4/3)-2\gamma\psi(1,1/3)-2\psi(0,1/3)\psi(1,1/3)+1\psi(2,4/3)\right) \tag{78}$$

$$\sum_{k=1}^{\infty}\frac{H(k)}{(2k+1)(3k+1)^3} = \frac{-1}{36}\left(-198-144\gamma\psi(0,1/2)-72\psi(0,1/2)^2+72\psi(1,3/2)+144\gamma\psi(0,1/3)+72\psi(0,1/3)^2\right.$$
$$-72\psi(1,4/3)+24\gamma\psi(1,1/3)+24\psi(0,1/3)\psi(1,1/3)-12\psi(2,4/3)+2\gamma\psi(2,1/3)$$
$$\left.+2\psi(0,1/3)\psi(2,1/3)+2\psi(1,1/3)\psi(1,1/3)-1\psi(3,4/3)\right) \tag{79}$$

$$\sum_{k=1}^{\infty}\frac{H(k)}{(2k+1)(3k+2)} = \frac{-1}{8}\left(-7+8\gamma\psi(0,1/2)+4\psi(0,1/2)^2-4\psi(1,3/2)-8\gamma\psi(0,2/3)-4\psi(0,2/3)^2\right.$$
$$\left.+4\psi(1,5/3)\right) \tag{80}$$

$$\sum_{k=1}^{\infty}\frac{H(k)}{(2k+1)(3k+2)^2} = \frac{-1}{24}\left(-15+48\gamma\psi(0,1/2)+24\psi(0,1/2)^2-24\psi(1,3/2)-48\gamma\psi(0,2/3)-24\psi(0,2/3)^2\right.$$
$$\left.+24\psi(1,5/3)+8\gamma\psi(1,2/3)+8\psi(0,2/3)\psi(1,2/3)-4\psi(2,5/3)\right) \tag{81}$$

$$\sum_{k=1}^{\infty}\frac{H(k)}{(2k+1)(3k+2)^3} = \frac{-1}{288}\left(-117+1152\gamma\psi(0,1/2)+576\psi(0,1/2)^2-576\psi(1,3/2)-1152\gamma\psi(0,2/3)\right.$$
$$-576\psi(0,2/3)^2+576\psi(1,5/3)+192\gamma\psi(1,2/3)+192\psi(0,2/3)\psi(1,2/3)-96\psi(2,5/3)$$
$$\left.-16\gamma\psi(2,2/3)-16\psi(0,2/3)\psi(2,2/3)-16\psi(1,2/3)\psi(1,2/3)+8\psi(3,5/3)\right) \tag{82}$$

$$\sum_{k=1}^{\infty}\frac{H(k)}{(2k+1)(4k+1)} = \frac{1}{4}\left(12+2\gamma\psi(0,1/2)+1\psi(0,1/2)^2-1\psi(1,3/2)-2\gamma\psi(0,1/4)-1\psi(0,1/4)^2\right.$$
$$\left.+1\psi(1,5/4)\right) \tag{83}$$

$$\sum_{k=1}^{\infty}\frac{H(k)}{(2k+1)(4k+1)^2} = \frac{-1}{16}\left(-80+8\gamma\psi(0,1/2)+4\psi(0,1/2)^2-4\psi(1,3/2)-8\gamma\psi(0,1/4)-4\psi(0,1/4)^2\right.$$
$$\left.+4\psi(1,5/4)-2\gamma\psi(1,1/4)-2\psi(0,1/4)\psi(1,1/4)+1\psi(2,5/4)\right) \tag{84}$$

$$\sum_{k=1}^{\infty}\frac{H(k)}{(2k+1)(4k+1)^3} = \frac{1}{128}\left(896+64\gamma\psi(0,1/2)+32\psi(0,1/2)^2-32\psi(1,3/2)-64\gamma\psi(0,1/4)-32\psi(0,1/4)^2\right.$$
$$+32\psi(1,5/4)-16\gamma\psi(1,1/4)-16\psi(0,1/4)\psi(1,1/4)+8\psi(2,5/4)-2\gamma\psi(2,1/4)$$
$$\left.-2\psi(0,1/4)\psi(2,1/4)-2\psi(1,1/4)\psi(1,1/4)+1\psi(3,5/4)\right) \tag{85}$$

$$\sum_{k=1}^{\infty}\frac{H(k)}{(2k+1)(4k+3)} = \frac{1}{36}\left(20-18\gamma\psi(0,1/2)-9\psi(0,1/2)^2+9\psi(1,3/2)+18\gamma\psi(0,3/4)+9\psi(0,3/4)^2\right.$$
$$\left.-9\psi(1,7/4)\right) \tag{86}$$

$$\sum_{k=1}^{\infty}\frac{H(k)}{(2k+1)(4k+3)^2} = \frac{-1}{432}\big(-112+216\gamma\psi(0,1/2)+108\psi(0,1/2)^2-108\psi(1,3/2)-216\gamma\psi(0,3/4)$$
$$-108\psi(0,3/4)^2+108\psi(1,7/4)+54\gamma\psi(1,3/4)+54\psi(0,3/4)\psi(1,3/4)$$
$$-27\psi(2,7/4)) \tag{87}$$

$$\sum_{k=1}^{\infty}\frac{H(k)}{(2k+1)(4k+3)^3} = \frac{-1}{1152}\big(-128+576\gamma\psi(0,1/2)+288\psi(0,1/2)^2-288\psi(1,3/2)-576\gamma\psi(0,3/4)$$
$$-288\psi(0,3/4)^2+288\psi(1,7/4)+144\gamma\psi(1,3/4)+144\psi(0,3/4)\psi(1,3/4)-72\psi(2,7/4)$$
$$-18\gamma\psi(2,3/4)-18\psi(0,3/4)\psi(2,3/4)-18\psi(1,3/4)\psi(1,3/4)+9\psi(3,7/4)) \tag{88}$$

$$\sum_{k=1}^{\infty}\frac{H(k)}{(2k+1)(5k+1)} = \frac{1}{6}\big(21+2\gamma\psi(0,1/2)+1\psi(0,1/2)^2-1\psi(1,3/2)-2\gamma\psi(0,1/5)-1\psi(0,1/5)^2$$
$$+1\psi(1,6/5)) \tag{89}$$

$$\sum_{k=1}^{\infty}\frac{H(k)}{(2k+1)(5k+1)^2} = \frac{-1}{90}\big(-540+20\gamma\psi(0,1/2)+10\psi(0,1/2)^2-10\psi(1,3/2)-20\gamma\psi(0,1/5)-10\psi(0,1/5)^2$$
$$+10\psi(1,6/5)-6\gamma\psi(1,1/5)-6\psi(0,1/5)\psi(1,1/5)+3\psi(2,6/5)) \tag{90}$$

$$\sum_{k=1}^{\infty}\frac{H(k)}{(2k+1)(5k+1)^3} = \frac{-1}{2700}\big(-22950-400\gamma\psi(0,1/2)-200\psi(0,1/2)^2+200\psi(1,3/2)+400\gamma\psi(0,1/5)$$
$$+200\psi(0,1/5)^2-200\psi(1,6/5)+120\gamma\psi(1,1/5)+120\psi(0,1/5)\psi(1,1/5)-60\psi(2,6/5)$$
$$+18\gamma\psi(2,1/5)+18\psi(0,1/5)\psi(2,1/5)+18\psi(1,1/5)\psi(1,1/5)-9\psi(3,6/5)) \tag{91}$$

$$\sum_{k=1}^{\infty}\frac{H(k)}{(2k+1)(5k+2)} = \frac{1}{8}\big(9+8\gamma\psi(0,1/2)+4\psi(0,1/2)^2-4\psi(1,3/2)-8\gamma\psi(0,2/5)-4\psi(0,2/5)^2$$
$$+4\psi(1,7/5)) \tag{92}$$

$$\sum_{k=1}^{\infty}\frac{H(k)}{(2k+1)(5k+2)^2} = \frac{-1}{40}\big(-35+80\gamma\psi(0,1/2)+40\psi(0,1/2)^2-40\psi(1,3/2)-80\gamma\psi(0,2/5)-40\psi(0,2/5)^2$$
$$+40\psi(1,7/5)-8\gamma\psi(1,2/5)-8\psi(0,2/5)\psi(1,2/5)+4\psi(2,7/5)) \tag{93}$$

$$\sum_{k=1}^{\infty}\frac{H(k)}{(2k+1)(5k+2)^3} = \frac{1}{800}\big(475+3200\gamma\psi(0,1/2)+1600\psi(0,1/2)^2-1600\psi(1,3/2)-3200\gamma\psi(0,2/5)$$
$$-1600\psi(0,2/5)^2+1600\psi(1,7/5)-320\gamma\psi(1,2/5)-320\psi(0,2/5)\psi(1,2/5)$$
$$+160\psi(2,7/5)-16\gamma\psi(2,2/5)-16\psi(0,2/5)\psi(2,2/5)-16\psi(1,2/5)\psi(1,2/5)$$
$$+8\psi(3,7/5)) \tag{94}$$

$$\sum_{k=1}^{\infty}\frac{H(k)}{(2k+1)(5k+3)} = \frac{-1}{18}\big(-11+18\gamma\psi(0,1/2)+9\psi(0,1/2)^2-9\psi(1,3/2)-18\gamma\psi(0,3/5)-9\psi(0,3/5)^2$$
$$+9\psi(1,8/5)) \tag{95}$$

$$\sum_{k=1}^{\infty}\frac{H(k)}{(2k+1)(5k+3)^2}=\frac{-1}{270}\big(-80+540\gamma\psi(0,1/2)+270\psi(0,1/2)^2-270\psi(1,3/2)-540\gamma\psi(0,3/5)$$
$$-270\psi(0,3/5)^2+270\psi(1,8/5)+54\gamma\psi(1,3/5)+54\psi(0,3/5)\psi(1,3/5)$$
$$-27\psi(2,8/5)) \tag{96}$$

$$\sum_{k=1}^{\infty}\frac{H(k)}{(2k+1)(5k+3)^3}=\frac{-1}{2700}\big(-350+10800\gamma\psi(0,1/2)+5400\psi(0,1/2)^2-5400\psi(1,3/2)-10800\gamma\psi(0,3/5)$$
$$-5400\psi(0,3/5)^2+5400\psi(1,8/5)+1080\gamma\psi(1,3/5)+1080\psi(0,3/5)\psi(1,3/5)$$
$$-540\psi(2,8/5)-54\gamma\psi(2,3/5)-54\psi(0,3/5)\psi(2,3/5)-54\psi(1,3/5)\psi(1,3/5)$$
$$+27\psi(3,8/5)) \tag{97}$$

$$\sum_{k=1}^{\infty}\frac{H(k)}{(2k+1)(5k+4)}=\frac{-1}{96}\big(-39+32\gamma\psi(0,1/2)+16\psi(0,1/2)^2-16\psi(1,3/2)-32\gamma\psi(0,4/5)-16\psi(0,4/5)^2$$
$$+16\psi(1,9/5)) \tag{98}$$

$$\sum_{k=1}^{\infty}\frac{H(k)}{(2k+1)(5k+4)^2}=\frac{1}{2880}\big(405-640\gamma\psi(0,1/2)-320\psi(0,1/2)^2+320\psi(1,3/2)+640\gamma\psi(0,4/5)$$
$$+320\psi(0,4/5)^2-320\psi(1,9/5)-192\gamma\psi(1,4/5)-192\psi(0,4/5)\psi(1,4/5)$$
$$+96\psi(2,9/5)) \tag{99}$$

$$\sum_{k=1}^{\infty}\frac{H(k)}{(2k+1)(5k+4)^3}=\frac{-1}{345600}\big(-15525+51200\gamma\psi(0,1/2)+25600\psi(0,1/2)^2-25600\psi(1,3/2)-51200\gamma\psi(0,4/5)$$
$$-25600\psi(0,4/5)^2+25600\psi(1,9/5)+15360\gamma\psi(1,4/5)+15360\psi(0,4/5)\psi(1,4/5)$$
$$-7680\psi(2,9/5)-2304\gamma\psi(2,4/5)-2304\psi(0,4/5)\psi(2,4/5)-2304\psi(1,4/5)\psi(1,4/5)$$
$$+1152\psi(3,9/5)) \tag{100}$$

$$\sum_{k=1}^{\infty}\frac{H(k)}{(2k+1)^2(3k+1)}=\frac{1}{4}\big(14+12\gamma\psi(0,1/2)+6\psi(0,1/2)^2-6\psi(1,3/2)-2\gamma\psi(1,1/2)-2\psi(0,1/2)\psi(1,1/2)$$
$$+1\psi(2,3/2)-12\gamma\psi(0,1/3)-6\psi(0,1/3)^2+6\psi(1,4/3)\big) \tag{101}$$

$$\sum_{k=1}^{\infty}\frac{H(k)}{(2k+1)^2(3k+1)^2}=\frac{1}{2}\big(10-24\gamma\psi(0,1/2)-12\psi(0,1/2)^2+12\psi(1,3/2)+2\gamma\psi(1,1/2)$$
$$+2\psi(0,1/2)\psi(1,1/2)-1\psi(2,3/2)+24\gamma\psi(0,1/3)+12\psi(0,1/3)^2-12\psi(1,4/3)$$
$$+2\gamma\psi(1,1/3)+2\psi(0,1/3)\psi(1,1/3)-1\psi(2,4/3)) \tag{102}$$

$$\sum_{k=1}^{\infty}\frac{H(k)}{(2k+1)^2(3k+1)^3}=\frac{1}{12}\big(78+432\gamma\psi(0,1/2)+216\psi(0,1/2)^2-216\psi(1,3/2)-24\gamma\psi(1,1/2)$$
$$-24\psi(0,1/2)\psi(1,1/2)+12\psi(2,3/2)-432\gamma\psi(0,1/3)-216\psi(0,1/3)^2+216\psi(1,4/3)$$
$$-48\gamma\psi(1,1/3)-48\psi(0,1/3)\psi(1,1/3)+24\psi(2,4/3)-2\gamma\psi(2,1/3)$$
$$-2\psi(0,1/3)\psi(2,1/3)-2\psi(1,1/3)\psi(1,1/3)+1\psi(3,4/3)) \tag{103}$$

$$\sum_{k=1}^{\infty}\frac{H(k)}{(2k+1)^2(3k+2)} = \frac{1}{8}\big(11+24\gamma\psi(0,1/2)+12\psi(0,1/2)^2-12\psi(1,3/2)+4\gamma\psi(1,1/2)+4\psi(0,1/2)\psi(1,1/2)$$
$$-2\psi(2,3/2)-24\gamma\psi(0,2/3)-12\psi(0,2/3)^2+12\psi(1,5/3)\big) \tag{104}$$

$$\sum_{k=1}^{\infty}\frac{H(k)}{(2k+1)^2(3k+2)^2} = \frac{-1}{8}\big(-7-96\gamma\psi(0,1/2)-48\psi(0,1/2)^2+48\psi(1,3/2)-8\gamma\psi(1,1/2)$$
$$-8\psi(0,1/2)\psi(1,1/2)+4\psi(2,3/2)+96\gamma\psi(0,2/3)+48\psi(0,2/3)^2-48\psi(1,5/3)$$
$$-8\gamma\psi(1,2/3)-8\psi(0,2/3)\psi(1,2/3)+4\psi(2,5/3)) \tag{105}$$

$$\sum_{k=1}^{\infty}\frac{H(k)}{(2k+1)^2(3k+2)^3} = \frac{-1}{96}\big(-51-3456\gamma\psi(0,1/2)-1728\psi(0,1/2)^2+1728\psi(1,3/2)-192\gamma\psi(1,1/2)$$
$$-192\psi(0,1/2)\psi(1,1/2)+96\psi(2,3/2)+3456\gamma\psi(0,2/3)+1728\psi(0,2/3)^2$$
$$-1728\psi(1,5/3)-384\gamma\psi(1,2/3)-384\psi(0,2/3)\psi(1,2/3)+192\psi(2,5/3)$$
$$+16\gamma\psi(2,2/3)+16\psi(0,2/3)\psi(2,2/3)+16\psi(1,2/3)\psi(1,2/3)-8\psi(3,5/3)) \tag{106}$$

$$\sum_{k=1}^{\infty}\frac{H(k)}{(2k+1)^2(4k+1)} = \frac{1}{8}\big(32+8\gamma\psi(0,1/2)+4\psi(0,1/2)^2-4\psi(1,3/2)-2\gamma\psi(1,1/2)-2\psi(0,1/2)\psi(1,1/2)$$
$$+1\psi(2,3/2)-8\gamma\psi(0,1/4)-4\psi(0,1/4)^2+4\psi(1,5/4)\big) \tag{107}$$

$$\sum_{k=1}^{\infty}\frac{H(k)}{(2k+1)^2(4k+1)^2} = \frac{1}{8}\big(48-16\gamma\psi(0,1/2)-8\psi(0,1/2)^2+8\psi(1,3/2)+2\gamma\psi(1,1/2)+2\psi(0,1/2)\psi(1,1/2)$$
$$-1\psi(2,3/2)+16\gamma\psi(0,1/4)+8\psi(0,1/4)^2-8\psi(1,5/4)+2\gamma\psi(1,1/4)$$
$$+2\psi(0,1/4)\psi(1,1/4)-1\psi(2,5/4)) \tag{108}$$

$$\sum_{k=1}^{\infty}\frac{H(k)}{(2k+1)^2(4k+1)^3} = \frac{1}{64}\big(512+192\gamma\psi(0,1/2)+96\psi(0,1/2)^2-96\psi(1,3/2)-16\gamma\psi(1,1/2)$$
$$-16\psi(0,1/2)\psi(1,1/2)+8\psi(2,3/2)-192\gamma\psi(0,1/4)-96\psi(0,1/4)^2+96\psi(1,5/4)$$
$$-32\gamma\psi(1,1/4)-32\psi(0,1/4)\psi(1,1/4)+16\psi(2,5/4)-2\gamma\psi(2,1/4)$$
$$-2\psi(0,1/4)\psi(2,1/4)-2\psi(1,1/4)\psi(1,1/4)+1\psi(3,5/4)) \tag{109}$$

$$\sum_{k=1}^{\infty}\frac{H(k)}{(2k+1)^2(4k+3)} = \frac{-1}{72}\big(-64-72\gamma\psi(0,1/2)-36\psi(0,1/2)^2+36\psi(1,3/2)-18\gamma\psi(1,1/2)$$
$$-18\psi(0,1/2)\psi(1,1/2)+9\psi(2,3/2)+72\gamma\psi(0,3/4)+36\psi(0,3/4)^2$$
$$-36\psi(1,7/4)) \tag{110}$$

$$\sum_{k=1}^{\infty}\frac{H(k)}{(2k+1)^2(4k+3)^2} = \frac{1}{216}\big(80+432\gamma\psi(0,1/2)+216\psi(0,1/2)^2-216\psi(1,3/2)+54\gamma\psi(1,1/2)$$
$$+54\psi(0,1/2)\psi(1,1/2)-27\psi(2,3/2)-432\gamma\psi(0,3/4)-216\psi(0,3/4)^2+216\psi(1,7/4)$$
$$+54\gamma\psi(1,3/4)+54\psi(0,3/4)\psi(1,3/4)-27\psi(2,7/4)) \tag{111}$$

$$\sum_{k=1}^{\infty}\frac{H(k)}{(2k+1)^2(4k+3)^3} = \frac{-1}{1728}\big(-256-5184\gamma\psi(0,1/2)-2592\psi(0,1/2)^2+2592\psi(1,3/2)-432\gamma\psi(1,1/2)$$
$$-432\psi(0,1/2)\psi(1,1/2)+216\psi(2,3/2)+5184\gamma\psi(0,3/4)+2592\psi(0,3/4)^2$$
$$-2592\psi(1,7/4)-864\gamma\psi(1,3/4)-864\psi(0,3/4)\psi(1,3/4)+432\psi(2,7/4)$$
$$+54\gamma\psi(2,3/4)+54\psi(0,3/4)\psi(2,3/4)+54\psi(1,3/4)\psi(1,3/4)-27\psi(3,7/4)) \tag{112}$$

$$\sum_{k=1}^{\infty}\frac{H(k)}{(2k+1)^2(5k+1)} = \frac{-1}{36}\big(-162-20\gamma\psi(0,1/2)-10\psi(0,1/2)^2+10\psi(1,3/2)+6\gamma\psi(1,1/2)$$
$$+6\psi(0,1/2)\psi(1,1/2)-3\psi(2,3/2)+20\gamma\psi(0,1/5)+10\psi(0,1/5)^2$$
$$-10\psi(1,6/5)) \tag{113}$$

$$\sum_{k=1}^{\infty}\frac{H(k)}{(2k+1)^2(5k+1)^2} = \frac{-1}{54}\big(-378+40\gamma\psi(0,1/2)+20\psi(0,1/2)^2-20\psi(1,3/2)-6\gamma\psi(1,1/2)$$
$$-6\psi(0,1/2)\psi(1,1/2)+3\psi(2,3/2)-40\gamma\psi(0,1/5)-20\psi(0,1/5)^2+20\psi(1,6/5)$$
$$-6\gamma\psi(1,1/5)-6\psi(0,1/5)\psi(1,1/5)+3\psi(2,6/5)) \tag{114}$$

$$\sum_{k=1}^{\infty}\frac{H(k)}{(2k+1)^2(5k+1)^3} = \frac{-1}{540}\big(-5130-400\gamma\psi(0,1/2)-200\psi(0,1/2)^2+200\psi(1,3/2)+40\gamma\psi(1,1/2)$$
$$+40\psi(0,1/2)\psi(1,1/2)-20\psi(2,3/2)+400\gamma\psi(0,1/5)+200\psi(0,1/5)^2-200\psi(1,6/5)$$
$$+80\gamma\psi(1,1/5)+80\psi(0,1/5)\psi(1,1/5)-40\psi(2,6/5)+6\gamma\psi(2,1/5)$$
$$+6\psi(0,1/5)\psi(2,1/5)+6\psi(1,1/5)\psi(1,1/5)-3\psi(3,6/5)) \tag{115}$$

$$\sum_{k=1}^{\infty}\frac{H(k)}{(2k+1)^2(5k+2)} = \frac{-1}{8}\big(-13-40\gamma\psi(0,1/2)-20\psi(0,1/2)^2+20\psi(1,3/2)+4\gamma\psi(1,1/2)$$
$$+4\psi(0,1/2)\psi(1,1/2)-2\psi(2,3/2)+40\gamma\psi(0,2/5)+20\psi(0,2/5)^2$$
$$-20\psi(1,7/5)) \tag{116}$$

$$\sum_{k=1}^{\infty}\frac{H(k)}{(2k+1)^2(5k+2)^2} = \frac{-1}{8}\big(-9+160\gamma\psi(0,1/2)+80\psi(0,1/2)^2-80\psi(1,3/2)-8\gamma\psi(1,1/2)$$
$$-8\psi(0,1/2)\psi(1,1/2)+4\psi(2,3/2)-160\gamma\psi(0,2/5)-80\psi(0,2/5)^2+80\psi(1,7/5)$$
$$-8\gamma\psi(1,2/5)-8\psi(0,2/5)\psi(1,2/5)+4\psi(2,7/5)) \tag{117}$$

$$\sum_{k=1}^{\infty}\frac{H(k)}{(2k+1)^2(5k+2)^3} = \frac{-1}{160}\big(-115-9600\gamma\psi(0,1/2)-4800\psi(0,1/2)^2+4800\psi(1,3/2)+320\gamma\psi(1,1/2)$$
$$+320\psi(0,1/2)\psi(1,1/2)-160\psi(2,3/2)+9600\gamma\psi(0,2/5)+4800\psi(0,2/5)^2$$
$$-4800\psi(1,7/5)+640\gamma\psi(1,2/5)+640\psi(0,2/5)\psi(1,2/5)-320\psi(2,7/5)$$
$$+16\gamma\psi(2,2/5)+16\psi(0,2/5)\psi(2,2/5)+16\psi(1,2/5)\psi(1,2/5)-8\psi(3,7/5)) \tag{118}$$

$$\sum_{k=1}^{\infty}\frac{H(k)}{(2k+1)^2(5k+3)} = \frac{1}{36}\big(34+180\gamma\psi(0,1/2)+90\psi(0,1/2)^2-90\psi(1,3/2)+18\gamma\psi(1,1/2)$$
$$+18\psi(0,1/2)\psi(1,1/2)-9\psi(2,3/2)-180\gamma\psi(0,3/5)-90\psi(0,3/5)^2$$
$$+90\psi(1,8/5)) \tag{119}$$

$$\sum_{k=1}^{\infty}\frac{H(k)}{(2k+1)^2(5k+3)^2}=\frac{-1}{54}\big(-22-1080\gamma\psi(0,1/2)-540\psi(0,1/2)^2+540\psi(1,3/2)-54\gamma\psi(1,1/2)$$
$$-54\psi(0,1/2)\psi(1,1/2)+27\psi(2,3/2)+1080\gamma\psi(0,3/5)+540\psi(0,3/5)^2-540\psi(1,8/5)$$
$$-54\gamma\psi(1,3/5)-54\psi(0,3/5)\psi(1,3/5)+27\psi(2,8/5)) \tag{120}$$

$$\sum_{k=1}^{\infty}\frac{H(k)}{(2k+1)^2(5k+3)^3}=\frac{-1}{60}\big(-10-3600\gamma\psi(0,1/2)-1800\psi(0,1/2)^2+1800\psi(1,3/2)-120\gamma\psi(1,1/2)$$
$$-120\psi(0,1/2)\psi(1,1/2)+60\psi(2,3/2)+3600\gamma\psi(0,3/5)+1800\psi(0,3/5)^2$$
$$-1800\psi(1,8/5)-240\gamma\psi(1,3/5)-240\psi(0,3/5)\psi(1,3/5)+120\psi(2,8/5)$$
$$+6\gamma\psi(2,3/5)+6\psi(0,3/5)\psi(2,3/5)+6\psi(1,3/5)\psi(1,3/5)-3\psi(3,8/5)) \tag{121}$$

$$\sum_{k=1}^{\infty}\frac{H(k)}{(2k+1)^2(5k+4)}=\frac{1}{288}\big(189+160\gamma\psi(0,1/2)+80\psi(0,1/2)^2-80\psi(1,3/2)+48\gamma\psi(1,1/2)$$
$$+48\psi(0,1/2)\psi(1,1/2)-24\psi(2,3/2)-160\gamma\psi(0,4/5)-80\psi(0,4/5)^2$$
$$+80\psi(1,9/5)) \tag{122}$$

$$\sum_{k=1}^{\infty}\frac{H(k)}{(2k+1)^2(5k+4)^2}=\frac{1}{1728}\big(351+1280\gamma\psi(0,1/2)+640\psi(0,1/2)^2-640\psi(1,3/2)+192\gamma\psi(1,1/2)$$
$$+192\psi(0,1/2)\psi(1,1/2)-96\psi(2,3/2)-1280\gamma\psi(0,4/5)-640\psi(0,4/5)^2+640\psi(1,9/5)$$
$$+192\gamma\psi(1,4/5)+192\psi(0,4/5)\psi(1,4/5)-96\psi(2,9/5)) \tag{123}$$

$$\sum_{k=1}^{\infty}\frac{H(k)}{(2k+1)^2(5k+4)^3}=\frac{-1}{69120}\big(-4185-51200\gamma\psi(0,1/2)-25600\psi(0,1/2)^2+25600\psi(1,3/2)-5120\gamma\psi(1,1/2)$$
$$-5120\psi(0,1/2)\psi(1,1/2)+2560\psi(2,3/2)+51200\gamma\psi(0,4/5)+25600\psi(0,4/5)^2$$
$$-25600\psi(1,9/5)-10240\gamma\psi(1,4/5)-10240\psi(0,4/5)\psi(1,4/5)+5120\psi(2,9/5)$$
$$+768\gamma\psi(2,4/5)+768\psi(0,4/5)\psi(2,4/5)+768\psi(1,4/5)\psi(1,4/5)$$
$$-384\psi(3,9/5)) \tag{124}$$

$$\sum_{k=1}^{\infty}\frac{H(k)}{(2k+1)^3(3k+1)}=\frac{-1}{8}\big(-84-72\gamma\psi(0,1/2)-36\psi(0,1/2)^2+36\psi(1,3/2)+12\gamma\psi(1,1/2)$$
$$+12\psi(0,1/2)\psi(1,1/2)-6\psi(2,3/2)-1\gamma\psi(2,1/2)-1\psi(0,1/2)\psi(2,1/2)$$
$$+1\psi(1,1/2)\psi(1,1/2)+72\gamma\psi(0,1/3)+36\psi(0,1/3)^2-36\psi(1,4/3)\big) \tag{125}$$

$$\sum_{k=1}^{\infty}\frac{H(k)}{(2k+1)^3(3k+1)^2}=\frac{1}{4}\big(-24-216\gamma\psi(0,1/2)-108\psi(0,1/2)^2+108\psi(1,3/2)+24\gamma\psi(1,1/2)$$
$$+24\psi(0,1/2)\psi(1,1/2)-12\psi(2,3/2)-1\gamma\psi(2,1/2)-1\psi(0,1/2)\psi(2,1/2)$$
$$+1\psi(1,1/2)\psi(1,1/2)+216\gamma\psi(0,1/3)+108\psi(0,1/3)^2-108\psi(1,4/3)$$
$$+12\gamma\psi(1,1/3)+12\psi(0,1/3)\psi(1,1/3)-6\psi(2,4/3)) \tag{126}$$

$$\sum_{k=1}^{\infty}\frac{H(k)}{(2k+1)^3(3k+1)^3} = \frac{-1}{4}\big(-126-864\gamma\psi(0,1/2)-432\psi(0,1/2)^2+432\psi(1,3/2)+72\gamma\psi(1,1/2)$$
$$+72\psi(0,1/2)\psi(1,1/2)-36\psi(2,3/2)-2\gamma\psi(2,1/2)-2\psi(0,1/2)\psi(2,1/2)$$
$$+2\psi(1,1/2)\psi(1,1/2)+864\gamma\psi(0,1/3)+432\psi(0,1/3)^2-432\psi(1,4/3)$$
$$+72\gamma\psi(1,1/3)+72\psi(0,1/3)\psi(1,1/3)-36\psi(2,4/3)+2\gamma\psi(2,1/3)$$
$$+2\psi(0,1/3)\psi(2,1/3)+2\psi(1,1/3)\psi(1,1/3)-1\psi(3,4/3)) \tag{127}$$

$$\sum_{k=1}^{\infty}\frac{H(k)}{(2k+1)^3(3k+2)} = \frac{-1}{8}\big(33+72\gamma\psi(0,1/2)+36\psi(0,1/2)^2-36\psi(1,3/2)+12\gamma\psi(1,1/2)$$
$$+12\psi(0,1/2)\psi(1,1/2)-6\psi(2,3/2)+1\gamma\psi(2,1/2)+1\psi(0,1/2)\psi(2,1/2)$$
$$-1\psi(1,1/2)\psi(1,1/2)-72\gamma\psi(0,2/3)-36\psi(0,2/3)^2+36\psi(1,5/3)\big) \tag{128}$$

$$\sum_{k=1}^{\infty}\frac{H(k)}{(2k+1)^3(3k+2)^2} = \frac{1}{8}\big(-87-432\gamma\psi(0,1/2)-216\psi(0,1/2)^2+216\psi(1,3/2)-48\gamma\psi(1,1/2)$$
$$-48\psi(0,1/2)\psi(1,1/2)+24\psi(2,3/2)-2\gamma\psi(2,1/2)-2\psi(0,1/2)\psi(2,1/2)$$
$$+2\psi(1,1/2)\psi(1,1/2)+432\gamma\psi(0,2/3)+216\psi(0,2/3)^2-216\psi(1,5/3)$$
$$-24\gamma\psi(1,2/3)-24\psi(0,2/3)\psi(1,2/3)+12\psi(2,5/3)) \tag{129}$$

$$\sum_{k=1}^{\infty}\frac{H(k)}{(2k+1)^3(3k+2)^3} = \frac{1}{32}\big(-747-6912\gamma\psi(0,1/2)-3456\psi(0,1/2)^2+3456\psi(1,3/2)-576\gamma\psi(1,1/2)$$
$$-576\psi(0,1/2)\psi(1,1/2)+288\psi(2,3/2)-16\gamma\psi(2,1/2)-16\psi(0,1/2)\psi(2,1/2)$$
$$+16\psi(1,1/2)\psi(1,1/2)+6912\gamma\psi(0,2/3)+3456\psi(0,2/3)^2-3456\psi(1,5/3)$$
$$-576\gamma\psi(1,2/3)-576\psi(0,2/3)\psi(1,2/3)+288\psi(2,5/3)+16\gamma\psi(2,2/3)$$
$$+16\psi(0,2/3)\psi(2,2/3)+16\psi(1,2/3)\psi(1,2/3)-8\psi(3,5/3)) \tag{130}$$

$$\sum_{k=1}^{\infty}\frac{H(k)}{(2k+1)^3(4k+1)} = \frac{-1}{16}\big(-128-32\gamma\psi(0,1/2)-16\psi(0,1/2)^2+16\psi(1,3/2)+8\gamma\psi(1,1/2)$$
$$+8\psi(0,1/2)\psi(1,1/2)-4\psi(2,3/2)-1\gamma\psi(2,1/2)-1\psi(0,1/2)\psi(2,1/2)$$
$$+1\psi(1,1/2)\psi(1,1/2)+32\gamma\psi(0,1/4)+16\psi(0,1/4)^2-16\psi(1,5/4)\big) \tag{131}$$

$$\sum_{k=1}^{\infty}\frac{H(k)}{(2k+1)^3(4k+1)^2} = \frac{-1}{16}\big(-64+96\gamma\psi(0,1/2)+48\psi(0,1/2)^2-48\psi(1,3/2)-16\gamma\psi(1,1/2)$$
$$-16\psi(0,1/2)\psi(1,1/2)+8\psi(2,3/2)+1\gamma\psi(2,1/2)+1\psi(0,1/2)\psi(2,1/2)$$
$$-1\psi(1,1/2)\psi(1,1/2)-96\gamma\psi(0,1/4)-48\psi(0,1/4)^2+48\psi(1,5/4)-8\gamma\psi(1,1/4)$$
$$-8\psi(0,1/4)\psi(1,1/4)+4\psi(2,5/4)) \tag{132}$$

$$\sum_{k=1}^{\infty}\frac{H(k)}{(2k+1)^3(4k+1)^3} = \frac{-1}{32}\big(-384-384\gamma\psi(0,1/2)-192\psi(0,1/2)^2+192\psi(1,3/2)+48\gamma\psi(1,1/2)$$
$$+48\psi(0,1/2)\psi(1,1/2)-24\psi(2,3/2)-2\gamma\psi(2,1/2)-2\psi(0,1/2)\psi(2,1/2)$$
$$+2\psi(1,1/2)\psi(1,1/2)+384\gamma\psi(0,1/4)+192\psi(0,1/4)^2-192\psi(1,5/4)$$
$$+48\gamma\psi(1,1/4)+48\psi(0,1/4)\psi(1,1/4)-24\psi(2,5/4)+2\gamma\psi(2,1/4)$$
$$+2\psi(0,1/4)\psi(2,1/4)+2\psi(1,1/4)\psi(1,1/4)-1\psi(3,5/4)) \tag{133}$$

$$\sum_{k=1}^{\infty}\frac{H(k)}{(2k+1)^3(4k+3)} = \frac{-1}{144}\big(256+288\gamma\psi(0,1/2)+144\psi(0,1/2)^2-144\psi(1,3/2)+72\gamma\psi(1,1/2)$$
$$+72\psi(0,1/2)\psi(1,1/2)-36\psi(2,3/2)+9\gamma\psi(2,1/2)+9\psi(0,1/2)\psi(2,1/2)$$
$$-9\psi(1,1/2)\psi(1,1/2)-288\gamma\psi(0,3/4)-144\psi(0,3/4)^2+144\psi(1,7/4)\big) \quad (134)$$

$$\sum_{k=1}^{\infty}\frac{H(k)}{(2k+1)^3(4k+3)^2} = \frac{-1}{432}\big(1088+2592\gamma\psi(0,1/2)+1296\psi(0,1/2)^2-1296\psi(1,3/2)+432\gamma\psi(1,1/2)$$
$$+432\psi(0,1/2)\psi(1,1/2)-216\psi(2,3/2)+27\gamma\psi(2,1/2)+27\psi(0,1/2)\psi(2,1/2)$$
$$-27\psi(1,1/2)\psi(1,1/2)-2592\gamma\psi(0,3/4)-1296\psi(0,3/4)^2+1296\psi(1,7/4)$$
$$+216\gamma\psi(1,3/4)+216\psi(0,3/4)\psi(1,3/4)-108\psi(2,7/4)) \quad (135)$$

$$\sum_{k=1}^{\infty}\frac{H(k)}{(2k+1)^3(4k+3)^3} = \frac{1}{864}\big(-2432-10368\gamma\psi(0,1/2)-5184\psi(0,1/2)^2+5184\psi(1,3/2)-1296\gamma\psi(1,1/2)$$
$$-1296\psi(0,1/2)\psi(1,1/2)+648\psi(2,3/2)-54\gamma\psi(2,1/2)-54\psi(0,1/2)\psi(2,1/2)$$
$$+54\psi(1,1/2)\psi(1,1/2)+10368\gamma\psi(0,3/4)+5184\psi(0,3/4)^2-5184\psi(1,7/4)$$
$$-1296\gamma\psi(1,3/4)-1296\psi(0,3/4)\psi(1,3/4)+648\psi(2,7/4)+54\gamma\psi(2,3/4)$$
$$+54\psi(0,3/4)\psi(2,3/4)+54\psi(1,3/4)\psi(1,3/4)-27\psi(3,7/4)) \quad (136)$$

$$\sum_{k=1}^{\infty}\frac{H(k)}{(2k+1)^3(5k+1)} = \frac{-1}{216}\big(-1620-200\gamma\psi(0,1/2)-100\psi(0,1/2)^2+100\psi(1,3/2)+60\gamma\psi(1,1/2)$$
$$+60\psi(0,1/2)\psi(1,1/2)-30\psi(2,3/2)-9\gamma\psi(2,1/2)-9\psi(0,1/2)\psi(2,1/2)$$
$$+9\psi(1,1/2)\psi(1,1/2)+200\gamma\psi(0,1/5)+100\psi(0,1/5)^2-100\psi(1,6/5)\big) \quad (137)$$

$$\sum_{k=1}^{\infty}\frac{H(k)}{(2k+1)^3(5k+1)^2} = \frac{1}{108}\big(720-200\gamma\psi(0,1/2)-100\psi(0,1/2)^2+100\psi(1,3/2)+40\gamma\psi(1,1/2)$$
$$+40\psi(0,1/2)\psi(1,1/2)-20\psi(2,3/2)-3\gamma\psi(2,1/2)-3\psi(0,1/2)\psi(2,1/2)$$
$$+3\psi(1,1/2)\psi(1,1/2)+200\gamma\psi(0,1/5)+100\psi(0,1/5)^2-100\psi(1,6/5)$$
$$+20\gamma\psi(1,1/5)+20\psi(0,1/5)\psi(1,1/5)-10\psi(2,6/5)) \quad (138)$$

$$\sum_{k=1}^{\infty}\frac{H(k)}{(2k+1)^3(5k+1)^3} = \frac{1}{324}\big(3690+800\gamma\psi(0,1/2)+400\psi(0,1/2)^2-400\psi(1,3/2)-120\gamma\psi(1,1/2)$$
$$-120\psi(0,1/2)\psi(1,1/2)+60\psi(2,3/2)+6\gamma\psi(2,1/2)+6\psi(0,1/2)\psi(2,1/2)$$
$$-6\psi(1,1/2)\psi(1,1/2)-800\gamma\psi(0,1/5)-400\psi(0,1/5)^2+400\psi(1,6/5)$$
$$-120\gamma\psi(1,1/5)-120\psi(0,1/5)\psi(1,1/5)+60\psi(2,6/5)-6\gamma\psi(2,1/5)$$
$$-6\psi(0,1/5)\psi(2,1/5)-6\psi(1,1/5)\psi(1,1/5)+3\psi(3,6/5)) \quad (139)$$

$$\sum_{k=1}^{\infty}\frac{H(k)}{(2k+1)^3(5k+2)} = \frac{-1}{8}\big(-65-200\gamma\psi(0,1/2)-100\psi(0,1/2)^2+100\psi(1,3/2)+20\gamma\psi(1,1/2)$$
$$+20\psi(0,1/2)\psi(1,1/2)-10\psi(2,3/2)-1\gamma\psi(2,1/2)-1\psi(0,1/2)\psi(2,1/2)$$
$$+1\psi(1,1/2)\psi(1,1/2)+200\gamma\psi(0,2/5)+100\psi(0,2/5)^2-100\psi(1,7/5)\big) \quad (140)$$

$$\sum_{k=1}^{\infty}\frac{H(k)}{(2k+1)^3(5k+2)^2}=\frac{1}{8}\big(-85-1200\gamma\psi(0,1/2)-600\psi(0,1/2)^2+600\psi(1,3/2)+80\gamma\psi(1,1/2)$$
$$+80\psi(0,1/2)\psi(1,1/2)-40\psi(2,3/2)-2\gamma\psi(2,1/2)-2\psi(0,1/2)\psi(2,1/2)$$
$$+2\psi(1,1/2)\psi(1,1/2)+1200\gamma\psi(0,2/5)+600\psi(0,2/5)^2-600\psi(1,7/5)$$
$$+40\gamma\psi(1,2/5)+40\psi(0,2/5)\psi(1,2/5)-20\psi(2,7/5)) \tag{141}$$

$$\sum_{k=1}^{\infty}\frac{H(k)}{(2k+1)^3(5k+2)^3}=\frac{-1}{32}\big(-795-19200\gamma\psi(0,1/2)-9600\psi(0,1/2)^2+9600\psi(1,3/2)+960\gamma\psi(1,1/2)$$
$$+960\psi(0,1/2)\psi(1,1/2)-480\psi(2,3/2)-16\gamma\psi(2,1/2)-16\psi(0,1/2)\psi(2,1/2)$$
$$+16\psi(1,1/2)\psi(1,1/2)+19200\gamma\psi(0,2/5)+9600\psi(0,2/5)^2-9600\psi(1,7/5)$$
$$+960\gamma\psi(1,2/5)+960\psi(0,2/5)\psi(1,2/5)-480\psi(2,7/5)+16\gamma\psi(2,2/5)$$
$$+16\psi(0,2/5)\psi(2,2/5)+16\psi(1,2/5)\psi(1,2/5)-8\psi(3,7/5)) \tag{142}$$

$$\sum_{k=1}^{\infty}\frac{H(k)}{(2k+1)^3(5k+3)}=\frac{-1}{72}\big(340+1800\gamma\psi(0,1/2)+900\psi(0,1/2)^2-900\psi(1,3/2)+180\gamma\psi(1,1/2)$$
$$+180\psi(0,1/2)\psi(1,1/2)-90\psi(2,3/2)+9\gamma\psi(2,1/2)+9\psi(0,1/2)\psi(2,1/2)$$
$$-9\psi(1,1/2)\psi(1,1/2)-1800\gamma\psi(0,3/5)-900\psi(0,3/5)^2+900\psi(1,8/5)\big) \tag{143}$$

$$\sum_{k=1}^{\infty}\frac{H(k)}{(2k+1)^3(5k+3)^2}=\frac{-1}{108}\big(1240+16200\gamma\psi(0,1/2)+8100\psi(0,1/2)^2-8100\psi(1,3/2)+1080\gamma\psi(1,1/2)$$
$$+1080\psi(0,1/2)\psi(1,1/2)-540\psi(2,3/2)+27\gamma\psi(2,1/2)+27\psi(0,1/2)\psi(2,1/2)$$
$$-27\psi(1,1/2)\psi(1,1/2)-16200\gamma\psi(0,3/5)-8100\psi(0,3/5)^2+8100\psi(1,8/5)$$
$$+540\gamma\psi(1,3/5)+540\psi(0,3/5)\psi(1,3/5)-270\psi(2,8/5)) \tag{144}$$

$$\sum_{k=1}^{\infty}\frac{H(k)}{(2k+1)^3(5k+3)^3}=\frac{1}{108}\big(-2570-64800\gamma\psi(0,1/2)-32400\psi(0,1/2)^2+32400\psi(1,3/2)-3240\gamma\psi(1,1/2)$$
$$-3240\psi(0,1/2)\psi(1,1/2)+1620\psi(2,3/2)-54\gamma\psi(2,1/2)-54\psi(0,1/2)\psi(2,1/2)$$
$$+54\psi(1,1/2)\psi(1,1/2)+64800\gamma\psi(0,3/5)+32400\psi(0,3/5)^2-32400\psi(1,8/5)$$
$$-3240\gamma\psi(1,3/5)-3240\psi(0,3/5)\psi(1,3/5)+1620\psi(2,8/5)+54\gamma\psi(2,3/5)$$
$$+54\psi(0,3/5)\psi(2,3/5)+54\psi(1,3/5)\psi(1,3/5)-27\psi(3,8/5)) \tag{145}$$

$$\sum_{k=1}^{\infty}\frac{H(k)}{(2k+1)^3(5k+4)}=\frac{-1}{864}\big(945+800\gamma\psi(0,1/2)+400\psi(0,1/2)^2-400\psi(1,3/2)+240\gamma\psi(1,1/2)$$
$$+240\psi(0,1/2)\psi(1,1/2)-120\psi(2,3/2)+36\gamma\psi(2,1/2)+36\psi(0,1/2)\psi(2,1/2)$$
$$-36\psi(1,1/2)\psi(1,1/2)-800\gamma\psi(0,4/5)-400\psi(0,4/5)^2+400\psi(1,9/5)\big) \tag{146}$$

$$\sum_{k=1}^{\infty}\frac{H(k)}{(2k+1)^3(5k+4)^2}=\frac{-1}{1728}\big(1845+3200\gamma\psi(0,1/2)+1600\psi(0,1/2)^2-1600\psi(1,3/2)+640\gamma\psi(1,1/2)$$
$$+640\psi(0,1/2)\psi(1,1/2)-320\psi(2,3/2)+48\gamma\psi(2,1/2)+48\psi(0,1/2)\psi(2,1/2)$$
$$-48\psi(1,1/2)\psi(1,1/2)-3200\gamma\psi(0,4/5)-1600\psi(0,4/5)^2+1600\psi(1,9/5)$$
$$+320\gamma\psi(1,4/5)+320\psi(0,4/5)\psi(1,4/5)-160\psi(2,9/5)) \tag{147}$$

$$\sum_{k=1}^{\infty}\frac{H(k)}{(2k+1)^3(5k+4)^3} = \frac{1}{41472}\big(-33705-102400\gamma\psi(0,1/2)-51200\psi(0,1/2)^2+51200\psi(1,3/2)-15360\gamma\psi(1,1/2)$$
$$-15360\psi(0,1/2)\psi(1,1/2)+7680\psi(2,3/2)-768\gamma\psi(2,1/2)-768\psi(0,1/2)\psi(2,1/2)$$
$$+768\psi(1,1/2)\psi(1,1/2)+102400\gamma\psi(0,4/5)+51200\psi(0,4/5)^2-51200\psi(1,9/5)$$
$$-15360\gamma\psi(1,4/5)-15360\psi(0,4/5)\psi(1,4/5)+7680\psi(2,9/5)+768\gamma\psi(2,4/5)$$
$$+768\psi(0,4/5)\psi(2,4/5)+768\psi(1,4/5)\psi(1,4/5)-384\psi(3,9/5)) \tag{148}$$

$$\sum_{k=1}^{\infty}\frac{H(k)}{(3k+1)(4k+1)} = \frac{-1}{2}\big(-7-2\gamma\psi(0,1/3)-1\psi(0,1/3)^2+1\psi(1,4/3)+2\gamma\psi(0,1/4)+1\psi(0,1/4)^2$$
$$-1\psi(1,5/4)) \tag{149}$$

$$\sum_{k=1}^{\infty}\frac{H(k)}{(3k+1)(4k+1)^2} = \frac{-1}{8}\big(-44+24\gamma\psi(0,1/3)+12\psi(0,1/3)^2-12\psi(1,4/3)-24\gamma\psi(0,1/4)-12\psi(0,1/4)^2$$
$$+12\psi(1,5/4)-2\gamma\psi(1,1/4)-2\psi(0,1/4)\psi(1,1/4)+1\psi(2,5/4)) \tag{150}$$

$$\sum_{k=1}^{\infty}\frac{H(k)}{(3k+1)(4k+1)^3} = \frac{1}{64}\big(480+576\gamma\psi(0,1/3)+288\psi(0,1/3)^2-288\psi(1,4/3)-576\gamma\psi(0,1/4)-288\psi(0,1/4)^2$$
$$+288\psi(1,5/4)-48\gamma\psi(1,1/4)-48\psi(0,1/4)\psi(1,1/4)+24\psi(2,5/4)-2\gamma\psi(2,1/4)$$
$$-2\psi(0,1/4)\psi(2,1/4)-2\psi(1,1/4)\psi(1,1/4)+1\psi(3,5/4)) \tag{151}$$

$$\sum_{k=1}^{\infty}\frac{H(k)}{(3k+1)(4k+3)} = \frac{-1}{90}\big(-65+18\gamma\psi(0,1/3)+9\psi(0,1/3)^2-9\psi(1,4/3)-18\gamma\psi(0,3/4)-9\psi(0,3/4)^2$$
$$+9\psi(1,7/4)) \tag{152}$$

$$\sum_{k=1}^{\infty}\frac{H(k)}{(3k+1)(4k+3)^2} = \frac{-1}{5400}\big(-1700+648\gamma\psi(0,1/3)+324\psi(0,1/3)^2-324\psi(1,4/3)-648\gamma\psi(0,3/4)$$
$$-324\psi(0,3/4)^2+324\psi(1,7/4)+270\gamma\psi(1,3/4)+270\psi(0,3/4)\psi(1,3/4)$$
$$-135\psi(2,7/4)) \tag{153}$$

$$\sum_{k=1}^{\infty}\frac{H(k)}{(3k+1)(4k+3)^3} = \frac{-1}{216000}\big(-28000+15552\gamma\psi(0,1/3)+7776\psi(0,1/3)^2-7776\psi(1,4/3)-15552\gamma\psi(0,3/4)$$
$$-7776\psi(0,3/4)^2+7776\psi(1,7/4)+6480\gamma\psi(1,3/4)+6480\psi(0,3/4)\psi(1,3/4)$$
$$-3240\psi(2,7/4)-1350\gamma\psi(2,3/4)-1350\psi(0,3/4)\psi(2,3/4)-1350\psi(1,3/4)\psi(1,3/4)$$
$$+675\psi(3,7/4)) \tag{154}$$

$$\sum_{k=1}^{\infty}\frac{H(k)}{(3k+1)(5k+1)} = \frac{-1}{4}\big(-16-2\gamma\psi(0,1/3)-1\psi(0,1/3)^2+1\psi(1,4/3)+2\gamma\psi(0,1/5)+1\psi(0,1/5)^2$$
$$-1\psi(1,6/5)) \tag{155}$$

$$\sum_{k=1}^{\infty}\frac{H(k)}{(3k+1)(5k+1)^2} = \frac{-1}{40}\big(-260+30\gamma\psi(0,1/3)+15\psi(0,1/3)^2-15\psi(1,4/3)-30\gamma\psi(0,1/5)-15\psi(0,1/5)^2$$
$$+15\psi(1,6/5)-4\gamma\psi(1,1/5)-4\psi(0,1/5)\psi(1,1/5)+2\psi(2,6/5)) \tag{156}$$

$$\sum_{k=1}^{\infty}\frac{H(k)}{(3k+1)(5k+1)^3} = \frac{1}{400}\big(3600+450\gamma\psi(0,1/3)+225\psi(0,1/3)^2-225\psi(1,4/3)-450\gamma\psi(0,1/5)$$
$$-225\psi(0,1/5)^2+225\psi(1,6/5)-60\gamma\psi(1,1/5)-60\psi(0,1/5)\psi(1,1/5)+30\psi(2,6/5)$$
$$-4\gamma\psi(2,1/5)-4\psi(0,1/5)\psi(2,1/5)-4\psi(1,1/5)\psi(1,1/5)+2\psi(3,6/5)) \quad (157)$$

$$\sum_{k=1}^{\infty}\frac{H(k)}{(3k+1)(5k+2)} = \frac{1}{8}\big(11-8\gamma\psi(0,1/3)-4\psi(0,1/3)^2+4\psi(1,4/3)+8\gamma\psi(0,2/5)+4\psi(0,2/5)^2$$
$$-4\psi(1,7/5)) \quad (158)$$

$$\sum_{k=1}^{\infty}\frac{H(k)}{(3k+1)(5k+2)^2} = \frac{1}{10}\big(10-30\gamma\psi(0,1/3)-15\psi(0,1/3)^2+15\psi(1,4/3)+30\gamma\psi(0,2/5)+15\psi(0,2/5)^2$$
$$-15\psi(1,7/5)-2\gamma\psi(1,2/5)-2\psi(0,2/5)\psi(1,2/5)+1\psi(2,7/5)) \quad (159)$$

$$\sum_{k=1}^{\infty}\frac{H(k)}{(3k+1)(5k+2)^3} = \frac{-1}{800}\big(-525+7200\gamma\psi(0,1/3)+3600\psi(0,1/3)^2-3600\psi(1,4/3)-7200\gamma\psi(0,2/5)$$
$$-3600\psi(0,2/5)^2+3600\psi(1,7/5)+480\gamma\psi(1,2/5)+480\psi(0,2/5)\psi(1,2/5)$$
$$-240\psi(2,7/5)-16\gamma\psi(2,2/5)-16\psi(0,2/5)\psi(2,2/5)-16\psi(1,2/5)\psi(1,2/5)$$
$$+8\psi(3,7/5)) \quad (160)$$

$$\sum_{k=1}^{\infty}\frac{H(k)}{(3k+1)(5k+3)} = \frac{-1}{72}\big(-56+18\gamma\psi(0,1/3)+9\psi(0,1/3)^2-9\psi(1,4/3)-18\gamma\psi(0,3/5)-9\psi(0,3/5)^2$$
$$+9\psi(1,8/5)) \quad (161)$$

$$\sum_{k=1}^{\infty}\frac{H(k)}{(3k+1)(5k+3)^2} = \frac{-1}{4320}\big(-1520+810\gamma\psi(0,1/3)+405\psi(0,1/3)^2-405\psi(1,4/3)-810\gamma\psi(0,3/5)$$
$$-405\psi(0,3/5)^2+405\psi(1,8/5)+216\gamma\psi(1,3/5)+216\psi(0,3/5)\psi(1,3/5)$$
$$-108\psi(2,8/5)) \quad (162)$$

$$\sum_{k=1}^{\infty}\frac{H(k)}{(3k+1)(5k+3)^3} = \frac{1}{86400}\big(12800-12150\gamma\psi(0,1/3)-6075\psi(0,1/3)^2+6075\psi(1,4/3)+12150\gamma\psi(0,3/5)$$
$$+6075\psi(0,3/5)^2-6075\psi(1,8/5)-3240\gamma\psi(1,3/5)-3240\psi(0,3/5)\psi(1,3/5)$$
$$+1620\psi(2,8/5)+432\gamma\psi(2,3/5)+432\psi(0,3/5)\psi(2,3/5)+432\psi(1,3/5)\psi(1,3/5)$$
$$-216\psi(3,8/5)) \quad (163)$$

$$\sum_{k=1}^{\infty}\frac{H(k)}{(3k+1)(5k+4)} = \frac{-1}{224}\big(-119+32\gamma\psi(0,1/3)+16\psi(0,1/3)^2-16\psi(1,4/3)-32\gamma\psi(0,4/5)-16\psi(0,4/5)^2$$
$$+16\psi(1,9/5)) \quad (164)$$

$$\sum_{k=1}^{\infty}\frac{H(k)}{(3k+1)(5k+4)^2} = \frac{1}{15680}\big(2695-960\gamma\psi(0,1/3)-480\psi(0,1/3)^2+480\psi(1,4/3)+960\gamma\psi(0,4/5)$$
$$+480\psi(0,4/5)^2-480\psi(1,9/5)-448\gamma\psi(1,4/5)-448\psi(0,4/5)\psi(1,4/5)$$
$$+224\psi(2,9/5)) \quad (165)$$

$$\sum_{k=1}^{\infty}\frac{H(k)}{(3k+1)(5k+4)^3}=\frac{-1}{4390400}\left(-231525+115200\gamma\psi(0,1/3)+57600\psi(0,1/3)^2-57600\psi(1,4/3)\right.$$
$$-115200\gamma\psi(0,4/5)-57600\psi(0,4/5)^2+57600\psi(1,9/5)+53760\gamma\psi(1,4/5)$$
$$+53760\psi(0,4/5)\psi(1,4/5)-26880\psi(2,9/5)-12544\gamma\psi(2,4/5)$$
$$\left.-12544\psi(0,4/5)\psi(2,4/5)-12544\psi(1,4/5)\psi(1,4/5)+6272\psi(3,9/5)\right) \quad (166)$$

$$\sum_{k=1}^{\infty}\frac{H(k)}{(3k+1)^2(4k+1)}=\frac{1}{6}\left(30+24\gamma\psi(0,1/3)+12\psi(0,1/3)^2-12\psi(1,4/3)-2\gamma\psi(1,1/3)-2\psi(0,1/3)\psi(1,1/3)\right.$$
$$\left.+1\psi(2,4/3)-24\gamma\psi(0,1/4)-12\psi(0,1/4)^2+12\psi(1,5/4)\right) \quad (167)$$

$$\sum_{k=1}^{\infty}\frac{H(k)}{(3k+1)^2(4k+1)^2}=\frac{1}{2}\left(14-48\gamma\psi(0,1/3)-24\psi(0,1/3)^2+24\psi(1,4/3)+2\gamma\psi(1,1/3)\right.$$
$$+2\psi(0,1/3)\psi(1,1/3)-1\psi(2,4/3)+48\gamma\psi(0,1/4)+24\psi(0,1/4)^2-24\psi(1,5/4)$$
$$\left.+2\gamma\psi(1,1/4)+2\psi(0,1/4)\psi(1,1/4)-1\psi(2,5/4)\right) \quad (168)$$

$$\sum_{k=1}^{\infty}\frac{H(k)}{(3k+1)^2(4k+1)^3}=\frac{-1}{16}\left(-144-1728\gamma\psi(0,1/3)-864\psi(0,1/3)^2+864\psi(1,4/3)+48\gamma\psi(1,1/3)\right.$$
$$+48\psi(0,1/3)\psi(1,1/3)-24\psi(2,4/3)+1728\gamma\psi(0,1/4)+864\psi(0,1/4)^2-864\psi(1,5/4)$$
$$+96\gamma\psi(1,1/4)+96\psi(0,1/4)\psi(1,1/4)-48\psi(2,5/4)+2\gamma\psi(2,1/4)$$
$$\left.+2\psi(0,1/4)\psi(2,1/4)+2\psi(1,1/4)\psi(1,1/4)-1\psi(3,5/4)\right) \quad (169)$$

$$\sum_{k=1}^{\infty}\frac{H(k)}{(3k+1)^2(4k+3)}=\frac{1}{450}\left(550+72\gamma\psi(0,1/3)+36\psi(0,1/3)^2-36\psi(1,4/3)+30\gamma\psi(1,1/3)\right.$$
$$+30\psi(0,1/3)\psi(1,1/3)-15\psi(2,4/3)-72\gamma\psi(0,3/4)-36\psi(0,3/4)^2$$
$$\left.+36\psi(1,7/4)\right) \quad (170)$$

$$\sum_{k=1}^{\infty}\frac{H(k)}{(3k+1)^2(4k+3)^2}=\frac{-1}{6750}\left(-3250-1296\gamma\psi(0,1/3)-648\psi(0,1/3)^2+648\psi(1,4/3)-270\gamma\psi(1,1/3)\right.$$
$$-270\psi(0,1/3)\psi(1,1/3)+135\psi(2,4/3)+1296\gamma\psi(0,3/4)+648\psi(0,3/4)^2$$
$$\left.-648\psi(1,7/4)-270\gamma\psi(1,3/4)-270\psi(0,3/4)\psi(1,3/4)+135\psi(2,7/4)\right) \quad (171)$$

$$\sum_{k=1}^{\infty}\frac{H(k)}{(3k+1)^2(4k+3)^3}=\frac{-1}{270000}\left(-50000-46656\gamma\psi(0,1/3)-23328\psi(0,1/3)^2+23328\psi(1,4/3)-6480\gamma\psi(1,1/3)\right.$$
$$-6480\psi(0,1/3)\psi(1,1/3)+3240\psi(2,4/3)+46656\gamma\psi(0,3/4)+23328\psi(0,3/4)^2$$
$$-23328\psi(1,7/4)-12960\gamma\psi(1,3/4)-12960\psi(0,3/4)\psi(1,3/4)+6480\psi(2,7/4)$$
$$+1350\gamma\psi(2,3/4)+1350\psi(0,3/4)\psi(2,3/4)+1350\psi(1,3/4)\psi(1,3/4)$$
$$\left.-675\psi(3,7/4)\right) \quad (172)$$

$$\sum_{k=1}^{\infty}\frac{H(k)}{(3k+1)^2(5k+1)}=\frac{-1}{24}\left(-132-30\gamma\psi(0,1/3)-15\psi(0,1/3)^2+15\psi(1,4/3)+4\gamma\psi(1,1/3)\right.$$
$$+4\psi(0,1/3)\psi(1,1/3)-2\psi(2,4/3)+30\gamma\psi(0,1/5)+15\psi(0,1/5)^2$$
$$\left.-15\psi(1,6/5)\right) \quad (173)$$

$$\sum_{k=1}^{\infty}\frac{H(k)}{(3k+1)^2(5k+1)^2} = \frac{-1}{8}\left(-64+30\gamma\psi(0,1/3)+15\psi(0,1/3)^2-15\psi(1,4/3)-2\gamma\psi(1,1/3)\right.$$
$$-2\psi(0,1/3)\psi(1,1/3)+1\psi(2,4/3)-30\gamma\psi(0,1/5)-15\psi(0,1/5)^2+15\psi(1,6/5)$$
$$\left.-2\gamma\psi(1,1/5)-2\psi(0,1/5)\psi(1,1/5)+1\psi(2,6/5)\right) \tag{174}$$

$$\sum_{k=1}^{\infty}\frac{H(k)}{(3k+1)^2(5k+1)^3} = \frac{-1}{160}\left(-1680-1350\gamma\psi(0,1/3)-675\psi(0,1/3)^2+675\psi(1,4/3)+60\gamma\psi(1,1/3)\right.$$
$$+60\psi(0,1/3)\psi(1,1/3)-30\psi(2,4/3)+1350\gamma\psi(0,1/5)+675\psi(0,1/5)^2-675\psi(1,6/5)$$
$$+120\gamma\psi(1,1/5)+120\psi(0,1/5)\psi(1,1/5)-60\psi(2,6/5)+4\gamma\psi(2,1/5)$$
$$\left.+4\psi(0,1/5)\psi(2,1/5)+4\psi(1,1/5)\psi(1,1/5)-2\psi(3,6/5)\right) \tag{175}$$

$$\sum_{k=1}^{\infty}\frac{H(k)}{(3k+1)^2(5k+2)} = \frac{1}{24}\left(51+120\gamma\psi(0,1/3)+60\psi(0,1/3)^2-60\psi(1,4/3)+8\gamma\psi(1,1/3)\right.$$
$$+8\psi(0,1/3)\psi(1,1/3)-4\psi(2,4/3)-120\gamma\psi(0,2/5)-60\psi(0,2/5)^2$$
$$\left.+60\psi(1,7/5)\right) \tag{176}$$

$$\sum_{k=1}^{\infty}\frac{H(k)}{(3k+1)^2(5k+2)^2} = \frac{-1}{8}\left(-11-240\gamma\psi(0,1/3)-120\psi(0,1/3)^2+120\psi(1,4/3)-8\gamma\psi(1,1/3)\right.$$
$$-8\psi(0,1/3)\psi(1,1/3)+4\psi(2,4/3)+240\gamma\psi(0,2/5)+120\psi(0,2/5)^2-120\psi(1,7/5)$$
$$\left.-8\gamma\psi(1,2/5)-8\psi(0,2/5)\psi(1,2/5)+4\psi(2,7/5)\right) \tag{177}$$

$$\sum_{k=1}^{\infty}\frac{H(k)}{(3k+1)^2(5k+2)^3} = \frac{1}{160}\left(135+21600\gamma\psi(0,1/3)+10800\psi(0,1/3)^2-10800\psi(1,4/3)+480\gamma\psi(1,1/3)\right.$$
$$+480\psi(0,1/3)\psi(1,1/3)-240\psi(2,4/3)-21600\gamma\psi(0,2/5)-10800\psi(0,2/5)^2$$
$$+10800\psi(1,7/5)+960\gamma\psi(1,2/5)+960\psi(0,2/5)\psi(1,2/5)-480\psi(2,7/5)$$
$$\left.-16\gamma\psi(2,2/5)-16\psi(0,2/5)\psi(2,2/5)-16\psi(1,2/5)\psi(1,2/5)+8\psi(3,7/5)\right) \tag{178}$$

$$\sum_{k=1}^{\infty}\frac{H(k)}{(3k+1)^2(5k+3)} = \frac{-1}{288}\left(-368-90\gamma\psi(0,1/3)-45\psi(0,1/3)^2+45\psi(1,4/3)-24\gamma\psi(1,1/3)\right.$$
$$-24\psi(0,1/3)\psi(1,1/3)+12\psi(2,4/3)+90\gamma\psi(0,3/5)+45\psi(0,3/5)^2$$
$$\left.-45\psi(1,8/5)\right) \tag{179}$$

$$\sum_{k=1}^{\infty}\frac{H(k)}{(3k+1)^2(5k+3)^2} = \frac{-1}{1728}\left(-896-810\gamma\psi(0,1/3)-405\psi(0,1/3)^2+405\psi(1,4/3)-108\gamma\psi(1,1/3)\right.$$
$$-108\psi(0,1/3)\psi(1,1/3)+54\psi(2,4/3)+810\gamma\psi(0,3/5)+405\psi(0,3/5)^2-405\psi(1,8/5)$$
$$\left.-108\gamma\psi(1,3/5)-108\psi(0,3/5)\psi(1,3/5)+54\psi(2,8/5)\right) \tag{180}$$

$$\sum_{k=1}^{\infty}\frac{H(k)}{(3k+1)^2(5k+3)^3} = \frac{1}{69120}\left(14080+36450\gamma\psi(0,1/3)+18225\psi(0,1/3)^2-18225\psi(1,4/3)+3240\gamma\psi(1,1/3)\right.$$
$$+3240\psi(0,1/3)\psi(1,1/3)-1620\psi(2,4/3)-36450\gamma\psi(0,3/5)-18225\psi(0,3/5)^2$$
$$+18225\psi(1,8/5)+6480\gamma\psi(1,3/5)+6480\psi(0,3/5)\psi(1,3/5)-3240\psi(2,8/5)$$
$$-432\gamma\psi(2,3/5)-432\psi(0,3/5)\psi(2,3/5)-432\psi(1,3/5)\psi(1,3/5)$$
$$\left.+216\psi(3,8/5)\right) \tag{181}$$

$$\sum_{k=1}^{\infty}\frac{H(k)}{(3k+1)^2(5k+4)} = \frac{1}{4704}\big(4263+480\gamma\psi(0,1/3)+240\psi(0,1/3)^2-240\psi(1,4/3)+224\gamma\psi(1,1/3)$$
$$+224\psi(0,1/3)\psi(1,1/3)-112\psi(2,4/3)-480\gamma\psi(0,4/5)-240\psi(0,4/5)^2$$
$$+240\psi(1,9/5)) \tag{182}$$

$$\sum_{k=1}^{\infty}\frac{H(k)}{(3k+1)^2(5k+4)^2} = \frac{1}{21952}\big(5831+1920\gamma\psi(0,1/3)+960\psi(0,1/3)^2-960\psi(1,4/3)+448\gamma\psi(1,1/3)$$
$$+448\psi(0,1/3)\psi(1,1/3)-224\psi(2,4/3)-1920\gamma\psi(0,4/5)-960\psi(0,4/5)^2$$
$$+960\psi(1,9/5)+448\gamma\psi(1,4/5)+448\psi(0,4/5)\psi(1,4/5)-224\psi(2,9/5)) \tag{183}$$

$$\sum_{k=1}^{\infty}\frac{H(k)}{(3k+1)^2(5k+4)^3} = \frac{-1}{6146560}\big(-468195-345600\gamma\psi(0,1/3)-172800\psi(0,1/3)^2+172800\psi(1,4/3)$$
$$-53760\gamma\psi(1,1/3)-53760\psi(0,1/3)\psi(1,1/3)+26880\psi(2,4/3)+345600\gamma\psi(0,4/5)$$
$$+172800\psi(0,4/5)^2-172800\psi(1,9/5)-107520\gamma\psi(1,4/5)-107520\psi(0,4/5)\psi(1,4/5)$$
$$+53760\psi(2,9/5)+12544\gamma\psi(2,4/5)+12544\psi(0,4/5)\psi(2,4/5)$$
$$+12544\psi(1,4/5)\psi(1,4/5)-6272\psi(3,9/5)) \tag{184}$$

$$\sum_{k=1}^{\infty}\frac{H(k)}{(3k+1)^3(4k+1)} = \frac{-1}{36}\big(-234-576\gamma\psi(0,1/3)-288\psi(0,1/3)^2+288\psi(1,4/3)+48\gamma\psi(1,1/3)$$
$$+48\psi(0,1/3)\psi(1,1/3)-24\psi(2,4/3)-2\gamma\psi(2,1/3)-2\psi(0,1/3)\psi(2,1/3)$$
$$-2\psi(1,1/3)\psi(1,1/3)+1\psi(3,4/3)+576\gamma\psi(0,1/4)+288\psi(0,1/4)^2$$
$$-288\psi(1,5/4)) \tag{185}$$

$$\sum_{k=1}^{\infty}\frac{H(k)}{(3k+1)^3(4k+1)^2} = \frac{-1}{12}\big(-102+1728\gamma\psi(0,1/3)+864\psi(0,1/3)^2-864\psi(1,4/3)-96\gamma\psi(1,1/3)$$
$$-96\psi(0,1/3)\psi(1,1/3)+48\psi(2,4/3)+2\gamma\psi(2,1/3)+2\psi(0,1/3)\psi(2,1/3)$$
$$+2\psi(1,1/3)\psi(1,1/3)-1\psi(3,4/3)-1728\gamma\psi(0,1/4)-864\psi(0,1/4)^2+864\psi(1,5/4)$$
$$-48\gamma\psi(1,1/4)-48\psi(0,1/4)\psi(1,1/4)+24\psi(2,5/4)) \tag{186}$$

$$\sum_{k=1}^{\infty}\frac{H(k)}{(3k+1)^3(4k+1)^3} = \frac{1}{4}\big(42+3456\gamma\psi(0,1/3)+1728\psi(0,1/3)^2-1728\psi(1,4/3)-144\gamma\psi(1,1/3)$$
$$-144\psi(0,1/3)\psi(1,1/3)+72\psi(2,4/3)+2\gamma\psi(2,1/3)+2\psi(0,1/3)\psi(2,1/3)$$
$$+2\psi(1,1/3)\psi(1,1/3)-1\psi(3,4/3)-3456\gamma\psi(0,1/4)-1728\psi(0,1/4)^2+1728\psi(1,5/4)$$
$$-144\gamma\psi(1,1/4)-144\psi(0,1/4)\psi(1,1/4)+72\psi(2,5/4)-2\gamma\psi(2,1/4)$$
$$-2\psi(0,1/4)\psi(2,1/4)-2\psi(1,1/4)\psi(1,1/4)+1\psi(3,5/4)) \tag{187}$$

$$\sum_{k=1}^{\infty}\frac{H(k)}{(3k+1)^3(4k+3)} = \frac{1}{4500}\big(7750-576\gamma\psi(0,1/3)-288\psi(0,1/3)^2+288\psi(1,4/3)-240\gamma\psi(1,1/3)$$
$$-240\psi(0,1/3)\psi(1,1/3)+120\psi(2,4/3)-50\gamma\psi(2,1/3)-50\psi(0,1/3)\psi(2,1/3)$$
$$-50\psi(1,1/3)\psi(1,1/3)+25\psi(3,4/3)+576\gamma\psi(0,3/4)+288\psi(0,3/4)^2$$
$$-288\psi(1,7/4)) \tag{188}$$

$$\sum_{k=1}^{\infty}\frac{H(k)}{(3k+1)^3(4k+3)^2}=\frac{1}{67500}\big(43750-15552\gamma\psi(0,1/3)-7776\psi(0,1/3)^2+7776\psi(1,4/3)-4320\gamma\psi(1,1/3)$$
$$-4320\psi(0,1/3)\psi(1,1/3)+2160\psi(2,4/3)-450\gamma\psi(2,1/3)-450\psi(0,1/3)\psi(2,1/3)$$
$$-450\psi(1,1/3)\psi(1,1/3)+225\psi(3,4/3)+15552\gamma\psi(0,3/4)+7776\psi(0,3/4)^2$$
$$-7776\psi(1,7/4)-2160\gamma\psi(1,3/4)-2160\psi(0,3/4)\psi(1,3/4)+1080\psi(2,7/4)) \tag{189}$$

$$\sum_{k=1}^{\infty}\frac{H(k)}{(3k+1)^3(4k+3)^3}=\frac{-1}{337500}\big(-81250+93312\gamma\psi(0,1/3)+46656\psi(0,1/3)^2-46656\psi(1,4/3)+19440\gamma\psi(1,1/3)$$
$$+19440\psi(0,1/3)\psi(1,1/3)-9720\psi(2,4/3)+1350\gamma\psi(2,1/3)+1350\psi(0,1/3)\psi(2,1/3)$$
$$+1350\psi(1,1/3)\psi(1,1/3)-675\psi(3,4/3)-93312\gamma\psi(0,3/4)-46656\psi(0,3/4)^2$$
$$+46656\psi(1,7/4)+19440\gamma\psi(1,3/4)+19440\psi(0,3/4)\psi(1,3/4)-9720\psi(2,7/4)$$
$$-1350\gamma\psi(2,3/4)-1350\psi(0,3/4)\psi(2,3/4)-1350\psi(1,3/4)\psi(1,3/4)$$
$$+675\psi(3,7/4)) \tag{190}$$

$$\sum_{k=1}^{\infty}\frac{H(k)}{(3k+1)^3(5k+1)}=\frac{-1}{144}\big(-1008-450\gamma\psi(0,1/3)-225\psi(0,1/3)^2+225\psi(1,4/3)+60\gamma\psi(1,1/3)$$
$$+60\psi(0,1/3)\psi(1,1/3)-30\psi(2,4/3)-4\gamma\psi(2,1/3)-4\psi(0,1/3)\psi(2,1/3)$$
$$-4\psi(1,1/3)\psi(1,1/3)+2\psi(3,4/3)+450\gamma\psi(0,1/5)+225\psi(0,1/5)^2$$
$$-225\psi(1,6/5)) \tag{191}$$

$$\sum_{k=1}^{\infty}\frac{H(k)}{(3k+1)^3(5k+1)^2}=\frac{1}{96}\big(912-1350\gamma\psi(0,1/3)-675\psi(0,1/3)^2+675\psi(1,4/3)+120\gamma\psi(1,1/3)$$
$$+120\psi(0,1/3)\psi(1,1/3)-60\psi(2,4/3)-4\gamma\psi(2,1/3)-4\psi(0,1/3)\psi(2,1/3)$$
$$-4\psi(1,1/3)\psi(1,1/3)+2\psi(3,4/3)+1350\gamma\psi(0,1/5)+675\psi(0,1/5)^2-675\psi(1,6/5)$$
$$+60\gamma\psi(1,1/5)+60\psi(0,1/5)\psi(1,1/5)-30\psi(2,6/5)) \tag{192}$$

$$\sum_{k=1}^{\infty}\frac{H(k)}{(3k+1)^3(5k+1)^3}=\frac{-1}{32}\big(-384-1350\gamma\psi(0,1/3)-675\psi(0,1/3)^2+675\psi(1,4/3)+90\gamma\psi(1,1/3)$$
$$+90\psi(0,1/3)\psi(1,1/3)-45\psi(2,4/3)-2\gamma\psi(2,1/3)-2\psi(0,1/3)\psi(2,1/3)$$
$$-2\psi(1,1/3)\psi(1,1/3)+1\psi(3,4/3)+1350\gamma\psi(0,1/5)+675\psi(0,1/5)^2-675\psi(1,6/5)$$
$$+90\gamma\psi(1,1/5)+90\psi(0,1/5)\psi(1,1/5)-45\psi(2,6/5)+2\gamma\psi(2,1/5)$$
$$+2\psi(0,1/5)\psi(2,1/5)+2\psi(1,1/5)\psi(1,1/5)-1\psi(3,6/5)) \tag{193}$$

$$\sum_{k=1}^{\infty}\frac{H(k)}{(3k+1)^3(5k+2)}=\frac{-1}{72}\big(-207+1800\gamma\psi(0,1/3)+900\psi(0,1/3)^2-900\psi(1,4/3)+120\gamma\psi(1,1/3)$$
$$+120\psi(0,1/3)\psi(1,1/3)-60\psi(2,4/3)+4\gamma\psi(2,1/3)+4\psi(0,1/3)\psi(2,1/3)$$
$$+4\psi(1,1/3)\psi(1,1/3)-2\psi(3,4/3)-1800\gamma\psi(0,2/5)-900\psi(0,2/5)^2$$
$$+900\psi(1,7/5)) \tag{194}$$

$$\sum_{k=1}^{\infty}\frac{H(k)}{(3k+1)^3(5k+2)^2}=\frac{1}{12}\big(21-2700\gamma\psi(0,1/3)-1350\psi(0,1/3)^2+1350\psi(1,4/3)-120\gamma\psi(1,1/3)$$
$$-120\psi(0,1/3)\psi(1,1/3)+60\psi(2,4/3)-2\gamma\psi(2,1/3)-2\psi(0,1/3)\psi(2,1/3)$$
$$-2\psi(1,1/3)\psi(1,1/3)+1\psi(3,4/3)+2700\gamma\psi(0,2/5)+1350\psi(0,2/5)^2-1350\psi(1,7/5)$$
$$-60\gamma\psi(1,2/5)-60\psi(0,2/5)\psi(1,2/5)+30\psi(2,7/5)) \tag{195}$$

$$\sum_{k=1}^{\infty}\frac{H(k)}{(3k+1)^3(5k+2)^3}=\frac{1}{32}\big(33-43200\gamma\psi(0,1/3)-21600\psi(0,1/3)^2+21600\psi(1,4/3)-1440\gamma\psi(1,1/3)$$
$$-1440\psi(0,1/3)\psi(1,1/3)+720\psi(2,4/3)-16\gamma\psi(2,1/3)-16\psi(0,1/3)\psi(2,1/3)$$
$$-16\psi(1,1/3)\psi(1,1/3)+8\psi(3,4/3)+43200\gamma\psi(0,2/5)+21600\psi(0,2/5)^2$$
$$-21600\psi(1,7/5)-1440\gamma\psi(1,2/5)-1440\psi(0,2/5)\psi(1,2/5)+720\psi(2,7/5)$$
$$+16\gamma\psi(2,2/5)+16\psi(0,2/5)\psi(2,2/5)+16\psi(1,2/5)\psi(1,2/5)-8\psi(3,7/5)) \tag{196}$$

$$\sum_{k=1}^{\infty}\frac{H(k)}{(3k+1)^3(5k+3)}=\frac{-1}{1152}\big(-2048+450\gamma\psi(0,1/3)+225\psi(0,1/3)^2-225\psi(1,4/3)+120\gamma\psi(1,1/3)$$
$$+120\psi(0,1/3)\psi(1,1/3)-60\psi(2,4/3)+16\gamma\psi(2,1/3)+16\psi(0,1/3)\psi(2,1/3)$$
$$+16\psi(1,1/3)\psi(1,1/3)-8\psi(3,4/3)-450\gamma\psi(0,3/5)-225\psi(0,3/5)^2$$
$$+225\psi(1,8/5)) \tag{197}$$

$$\sum_{k=1}^{\infty}\frac{H(k)}{(3k+1)^3(5k+3)^2}=\frac{1}{13824}\big(9472-12150\gamma\psi(0,1/3)-6075\psi(0,1/3)^2+6075\psi(1,4/3)-2160\gamma\psi(1,1/3)$$
$$-2160\psi(0,1/3)\psi(1,1/3)+1080\psi(2,4/3)-144\gamma\psi(2,1/3)-144\psi(0,1/3)\psi(2,1/3)$$
$$-144\psi(1,1/3)\psi(1,1/3)+72\psi(3,4/3)+12150\gamma\psi(0,3/5)+6075\psi(0,3/5)^2$$
$$-6075\psi(1,8/5)-1080\gamma\psi(1,3/5)-1080\psi(0,3/5)\psi(1,3/5)+540\psi(2,8/5)) \tag{198}$$

$$\sum_{k=1}^{\infty}\frac{H(k)}{(3k+1)^3(5k+3)^3}=\frac{1}{27648}\big(7168-36450\gamma\psi(0,1/3)-18225\psi(0,1/3)^2+18225\psi(1,4/3)-4860\gamma\psi(1,1/3)$$
$$-4860\psi(0,1/3)\psi(1,1/3)+2430\psi(2,4/3)-216\gamma\psi(2,1/3)-216\psi(0,1/3)\psi(2,1/3)$$
$$-216\psi(1,1/3)\psi(1,1/3)+108\psi(3,4/3)+36450\gamma\psi(0,3/5)+18225\psi(0,3/5)^2$$
$$-18225\psi(1,8/5)-4860\gamma\psi(1,3/5)-4860\psi(0,3/5)\psi(1,3/5)+2430\psi(2,8/5)$$
$$+216\gamma\psi(2,3/5)+216\psi(0,3/5)\psi(2,3/5)+216\psi(1,3/5)\psi(1,3/5)$$
$$-108\psi(3,8/5)) \tag{199}$$

$$\sum_{k=1}^{\infty}\frac{H(k)}{(3k+1)^3(5k+4)}=\frac{1}{98784}\big(126567-7200\gamma\psi(0,1/3)-3600\psi(0,1/3)^2+3600\psi(1,4/3)-3360\gamma\psi(1,1/3)$$
$$-3360\psi(0,1/3)\psi(1,1/3)+1680\psi(2,4/3)-784\gamma\psi(2,1/3)-784\psi(0,1/3)\psi(2,1/3)$$
$$-784\psi(1,1/3)\psi(1,1/3)+392\psi(3,4/3)+7200\gamma\psi(0,4/5)+3600\psi(0,4/5)^2$$
$$-3600\psi(1,9/5)) \tag{200}$$

$$\sum_{k=1}^{\infty}\frac{H(k)}{(3k+1)^3(5k+4)^2}=\frac{-1}{460992}\left(-165669+43200\gamma\psi(0,1/3)+21600\psi(0,1/3)^2-21600\psi(1,4/3)+13440\gamma\psi(1,1/3)\right.$$
$$+13440\psi(0,1/3)\psi(1,1/3)-6720\psi(2,4/3)+1568\gamma\psi(2,1/3)+1568\psi(0,1/3)\psi(2,1/3)$$
$$+1568\psi(1,1/3)\psi(1,1/3)-784\psi(3,4/3)-43200\gamma\psi(0,4/5)-21600\psi(0,4/5)^2$$
$$\left.+21600\psi(1,9/5)+6720\gamma\psi(1,4/5)+6720\psi(0,4/5)\psi(1,4/5)-3360\psi(2,9/5)\right) \tag{201}$$

$$\sum_{k=1}^{\infty}\frac{H(k)}{(3k+1)^3(5k+4)^3}=\frac{1}{8605184}\left(857157-691200\gamma\psi(0,1/3)-345600\psi(0,1/3)^2+345600\psi(1,4/3)\right.$$
$$-161280\gamma\psi(1,1/3)-161280\psi(0,1/3)\psi(1,1/3)+80640\psi(2,4/3)-12544\gamma\psi(2,1/3)$$
$$-12544\psi(0,1/3)\psi(2,1/3)-12544\psi(1,1/3)\psi(1,1/3)+6272\psi(3,4/3)$$
$$+691200\gamma\psi(0,4/5)+345600\psi(0,4/5)^2-345600\psi(1,9/5)-161280\gamma\psi(1,4/5)$$
$$-161280\psi(0,4/5)\psi(1,4/5)+80640\psi(2,9/5)+12544\gamma\psi(2,4/5)$$
$$\left.+12544\psi(0,4/5)\psi(2,4/5)+12544\psi(1,4/5)\psi(1,4/5)-6272\psi(3,9/5)\right) \tag{202}$$

$$\sum_{k=1}^{\infty}\frac{H(k)}{(3k+2)(4k+1)}=\frac{1}{40}\left(55+8\gamma\psi(0,2/3)+4\psi(0,2/3)^2-4\psi(1,5/3)-8\gamma\psi(0,1/4)-4\psi(0,1/4)^2\right.$$
$$\left.+4\psi(1,5/4)\right) \tag{203}$$

$$\sum_{k=1}^{\infty}\frac{H(k)}{(3k+2)(4k+1)^2}=\frac{-1}{200}\left(-475+24\gamma\psi(0,2/3)+12\psi(0,2/3)^2-12\psi(1,5/3)-24\gamma\psi(0,1/4)-12\psi(0,1/4)^2\right.$$
$$\left.+12\psi(1,5/4)-10\gamma\psi(1,1/4)-10\psi(0,1/4)\psi(1,1/4)+5\psi(2,5/4)\right) \tag{204}$$

$$\sum_{k=1}^{\infty}\frac{H(k)}{(3k+2)(4k+1)^3}=\frac{1}{8000}\left(27000+576\gamma\psi(0,2/3)+288\psi(0,2/3)^2-288\psi(1,5/3)-576\gamma\psi(0,1/4)\right.$$
$$-288\psi(0,1/4)^2+288\psi(1,5/4)-240\gamma\psi(1,1/4)-240\psi(0,1/4)\psi(1,1/4)+120\psi(2,5/4)$$
$$\left.-50\gamma\psi(2,1/4)-50\psi(0,1/4)\psi(2,1/4)-50\psi(1,1/4)\psi(1,1/4)+25\psi(3,5/4)\right) \tag{205}$$

$$\sum_{k=1}^{\infty}\frac{H(k)}{(3k+2)(4k+3)}=\frac{1}{72}\left(17-72\gamma\psi(0,2/3)-36\psi(0,2/3)^2+36\psi(1,5/3)+72\gamma\psi(0,3/4)+36\psi(0,3/4)^2\right.$$
$$\left.-36\psi(1,7/4)\right) \tag{206}$$

$$\sum_{k=1}^{\infty}\frac{H(k)}{(3k+2)(4k+3)^2}=\frac{-1}{216}\left(-25+648\gamma\psi(0,2/3)+324\psi(0,2/3)^2-324\psi(1,5/3)-648\gamma\psi(0,3/4)\right.$$
$$-324\psi(0,3/4)^2+324\psi(1,7/4)+54\gamma\psi(1,3/4)+54\psi(0,3/4)\psi(1,3/4)$$
$$\left.-27\psi(2,7/4)\right) \tag{207}$$

$$\sum_{k=1}^{\infty}\frac{H(k)}{(3k+2)(4k+3)^3}=\frac{1}{1728}\left(88-15552\gamma\psi(0,2/3)-7776\psi(0,2/3)^2+7776\psi(1,5/3)+15552\gamma\psi(0,3/4)\right.$$
$$+7776\psi(0,3/4)^2-7776\psi(1,7/4)-1296\gamma\psi(1,3/4)-1296\psi(0,3/4)\psi(1,3/4)$$
$$+648\psi(2,7/4)+54\gamma\psi(2,3/4)+54\psi(0,3/4)\psi(2,3/4)+54\psi(1,3/4)\psi(1,3/4)$$
$$\left.-27\psi(3,7/4)\right) \tag{208}$$

$$\sum_{k=1}^{\infty}\frac{H(k)}{(3k+2)(5k+1)} = \frac{-1}{56}\left(-91-8\gamma\psi(0,2/3)-4\psi(0,2/3)^2+4\psi(1,5/3)+8\gamma\psi(0,1/5)+4\psi(0,1/5)^2\right.$$
$$\left.-4\psi(1,6/5)\right) \tag{209}$$

$$\sum_{k=1}^{\infty}\frac{H(k)}{(3k+2)(5k+1)^2} = \frac{-1}{1960}\left(-5635+120\gamma\psi(0,2/3)+60\psi(0,2/3)^2-60\psi(1,5/3)-120\gamma\psi(0,1/5)\right.$$
$$-60\psi(0,1/5)^2+60\psi(1,6/5)-56\gamma\psi(1,1/5)-56\psi(0,1/5)\psi(1,1/5)$$
$$\left.+28\psi(2,6/5)\right) \tag{210}$$

$$\sum_{k=1}^{\infty}\frac{H(k)}{(3k+2)(5k+1)^3} = \frac{-1}{68600}\left(-282975-1800\gamma\psi(0,2/3)-900\psi(0,2/3)^2+900\psi(1,5/3)+1800\gamma\psi(0,1/5)\right.$$
$$+900\psi(0,1/5)^2-900\psi(1,6/5)+840\gamma\psi(1,1/5)+840\psi(0,1/5)\psi(1,1/5)-420\psi(2,6/5)$$
$$\left.+196\gamma\psi(2,1/5)+196\psi(0,1/5)\psi(2,1/5)+196\psi(1,1/5)\psi(1,1/5)-98\psi(3,6/5)\right) \tag{211}$$

$$\sum_{k=1}^{\infty}\frac{H(k)}{(3k+2)(5k+2)} = \frac{-1}{8}\left(-4-2\gamma\psi(0,2/3)-1\psi(0,2/3)^2+1\psi(1,5/3)+2\gamma\psi(0,2/5)+1\psi(0,2/5)^2\right.$$
$$\left.-1\psi(1,7/5)\right) \tag{212}$$

$$\sum_{k=1}^{\infty}\frac{H(k)}{(3k+2)(5k+2)^2} = \frac{-1}{160}\left(-65+30\gamma\psi(0,2/3)+15\psi(0,2/3)^2-15\psi(1,5/3)-30\gamma\psi(0,2/5)-15\psi(0,2/5)^2\right.$$
$$\left.+15\psi(1,7/5)-8\gamma\psi(1,2/5)-8\psi(0,2/5)\psi(1,2/5)+4\psi(2,7/5)\right) \tag{213}$$

$$\sum_{k=1}^{\infty}\frac{H(k)}{(3k+2)(5k+2)^3} = \frac{-1}{3200}\left(-900-450\gamma\psi(0,2/3)-225\psi(0,2/3)^2+225\psi(1,5/3)+450\gamma\psi(0,2/5)\right.$$
$$+225\psi(0,2/5)^2-225\psi(1,7/5)+120\gamma\psi(1,2/5)+120\psi(0,2/5)\psi(1,2/5)-60\psi(2,7/5)$$
$$\left.+16\gamma\psi(2,2/5)+16\psi(0,2/5)\psi(2,2/5)+16\psi(1,2/5)\psi(1,2/5)-8\psi(3,7/5)\right) \tag{214}$$

$$\sum_{k=1}^{\infty}\frac{H(k)}{(3k+2)(5k+3)} = \frac{1}{72}\left(19+72\gamma\psi(0,2/3)+36\psi(0,2/3)^2-36\psi(1,5/3)-72\gamma\psi(0,3/5)-36\psi(0,3/5)^2\right.$$
$$\left.+36\psi(1,8/5)\right) \tag{215}$$

$$\sum_{k=1}^{\infty}\frac{H(k)}{(3k+2)(5k+3)^2} = \frac{1}{1080}\left(145-3240\gamma\psi(0,2/3)-1620\psi(0,2/3)^2+1620\psi(1,5/3)+3240\gamma\psi(0,3/5)\right.$$
$$+1620\psi(0,3/5)^2-1620\psi(1,8/5)+216\gamma\psi(1,3/5)+216\psi(0,3/5)\psi(1,3/5)$$
$$\left.-108\psi(2,8/5)\right) \tag{216}$$

$$\sum_{k=1}^{\infty}\frac{H(k)}{(3k+2)(5k+3)^3} = \frac{1}{5400}\left(325+48600\gamma\psi(0,2/3)+24300\psi(0,2/3)^2-24300\psi(1,5/3)-48600\gamma\psi(0,3/5)\right.$$
$$-24300\psi(0,3/5)^2+24300\psi(1,8/5)-3240\gamma\psi(1,3/5)-3240\psi(0,3/5)\psi(1,3/5)$$
$$+1620\psi(2,8/5)-108\gamma\psi(2,3/5)-108\psi(0,3/5)\psi(2,3/5)-108\psi(1,3/5)\psi(1,3/5)$$
$$\left.+54\psi(3,8/5)\right) \tag{217}$$

$$\sum_{k=1}^{\infty}\frac{H(k)}{(3k+2)(5k+4)} = \frac{1}{64}\left(11-32\gamma\psi(0,2/3)-16\psi(0,2/3)^2+16\psi(1,5/3)+32\gamma\psi(0,4/5)+16\psi(0,4/5)^2\right.$$
$$\left.-16\psi(1,9/5)\right) \tag{218}$$

$$\sum_{k=1}^{\infty}\frac{H(k)}{(3k+2)(5k+4)^2} = \frac{-1}{80}\left(-5+60\gamma\psi(0,2/3)+30\psi(0,2/3)^2-30\psi(1,5/3)-60\gamma\psi(0,4/5)-30\psi(0,4/5)^2\right.$$
$$\left.+30\psi(1,9/5)+8\gamma\psi(1,4/5)+8\psi(0,4/5)\psi(1,4/5)-4\psi(2,9/5)\right) \tag{219}$$

$$\sum_{k=1}^{\infty}\frac{H(k)}{(3k+2)(5k+4)^3} = \frac{-1}{25600}\left(-525+28800\gamma\psi(0,2/3)+14400\psi(0,2/3)^2-14400\psi(1,5/3)-28800\gamma\psi(0,4/5)\right.$$
$$-14400\psi(0,4/5)^2+14400\psi(1,9/5)+3840\gamma\psi(1,4/5)+3840\psi(0,4/5)\psi(1,4/5)$$
$$-1920\psi(2,9/5)-256\gamma\psi(2,4/5)-256\psi(0,4/5)\psi(2,4/5)-256\psi(1,4/5)\psi(1,4/5)$$
$$\left.+128\psi(3,9/5)\right) \tag{220}$$

$$\sum_{k=1}^{\infty}\frac{H(k)}{(3k+2)^2(4k+1)} = \frac{-1}{600}\left(-525-96\gamma\psi(0,2/3)-48\psi(0,2/3)^2+48\psi(1,5/3)+40\gamma\psi(1,2/3)\right.$$
$$+40\psi(0,2/3)\psi(1,2/3)-20\psi(2,5/3)+96\gamma\psi(0,1/4)+48\psi(0,1/4)^2$$
$$\left.-48\psi(1,5/4)\right) \tag{221}$$

$$\sum_{k=1}^{\infty}\frac{H(k)}{(3k+2)^2(4k+1)^2} = \frac{-1}{1000}\left(-1375+192\gamma\psi(0,2/3)+96\psi(0,2/3)^2-96\psi(1,5/3)-40\gamma\psi(1,2/3)\right.$$
$$-40\psi(0,2/3)\psi(1,2/3)+20\psi(2,5/3)-192\gamma\psi(0,1/4)-96\psi(0,1/4)^2+96\psi(1,5/4)$$
$$\left.-40\gamma\psi(1,1/4)-40\psi(0,1/4)\psi(1,1/4)+20\psi(2,5/4)\right) \tag{222}$$

$$\sum_{k=1}^{\infty}\frac{H(k)}{(3k+2)^2(4k+1)^3} = \frac{1}{10000}\left(18750+1728\gamma\psi(0,2/3)+864\psi(0,2/3)^2-864\psi(1,5/3)-240\gamma\psi(1,2/3)\right.$$
$$-240\psi(0,2/3)\psi(1,2/3)+120\psi(2,5/3)-1728\gamma\psi(0,1/4)-864\psi(0,1/4)^2$$
$$+864\psi(1,5/4)-480\gamma\psi(1,1/4)-480\psi(0,1/4)\psi(1,1/4)+240\psi(2,5/4)$$
$$\left.-50\gamma\psi(2,1/4)-50\psi(0,1/4)\psi(2,1/4)-50\psi(1,1/4)\psi(1,1/4)+25\psi(3,5/4)\right) \tag{223}$$

$$\sum_{k=1}^{\infty}\frac{H(k)}{(3k+2)^2(4k+3)} = \frac{-1}{72}\left(-13-288\gamma\psi(0,2/3)-144\psi(0,2/3)^2+144\psi(1,5/3)-24\gamma\psi(1,2/3)\right.$$
$$-24\psi(0,2/3)\psi(1,2/3)+12\psi(2,5/3)+288\gamma\psi(0,3/4)+144\psi(0,3/4)^2$$
$$\left.-144\psi(1,7/4)\right) \tag{224}$$

$$\sum_{k=1}^{\infty}\frac{H(k)}{(3k+2)^2(4k+3)^2} = \frac{1}{216}\left(17+5184\gamma\psi(0,2/3)+2592\psi(0,2/3)^2-2592\psi(1,5/3)+216\gamma\psi(1,2/3)\right.$$
$$+216\psi(0,2/3)\psi(1,2/3)-108\psi(2,5/3)-5184\gamma\psi(0,3/4)-2592\psi(0,3/4)^2$$
$$\left.+2592\psi(1,7/4)+216\gamma\psi(1,3/4)+216\psi(0,3/4)\psi(1,3/4)-108\psi(2,7/4)\right) \tag{225}$$

$$\sum_{k=1}^{\infty}\frac{H(k)}{(3k+2)^2(4k+3)^3}=\frac{-1}{432}\left(-14-46656\gamma\psi(0,2/3)-23328\psi(0,2/3)^2+23328\psi(1,5/3)-1296\gamma\psi(1,2/3)\right.$$
$$-1296\psi(0,2/3)\psi(1,2/3)+648\psi(2,5/3)+46656\gamma\psi(0,3/4)+23328\psi(0,3/4)^2$$
$$-23328\psi(1,7/4)-2592\gamma\psi(1,3/4)-2592\psi(0,3/4)\psi(1,3/4)+1296\psi(2,7/4)$$
$$\left.+54\gamma\psi(2,3/4)+54\psi(0,3/4)\psi(2,3/4)+54\psi(1,3/4)\psi(1,3/4)-27\psi(3,7/4)\right) \tag{226}$$

$$\sum_{k=1}^{\infty}\frac{H(k)}{(3k+2)^2(5k+1)}=\frac{1}{294}\left(294+30\gamma\psi(0,2/3)+15\psi(0,2/3)^2-15\psi(1,5/3)-14\gamma\psi(1,2/3)\right.$$
$$-14\psi(0,2/3)\psi(1,2/3)+7\psi(2,5/3)-30\gamma\psi(0,1/5)-15\psi(0,1/5)^2$$
$$\left.+15\psi(1,6/5)\right) \tag{227}$$

$$\sum_{k=1}^{\infty}\frac{H(k)}{(3k+2)^2(5k+1)^2}=\frac{1}{2744}\left(4459-240\gamma\psi(0,2/3)-120\psi(0,2/3)^2+120\psi(1,5/3)+56\gamma\psi(1,2/3)\right.$$
$$+56\psi(0,2/3)\psi(1,2/3)-28\psi(2,5/3)+240\gamma\psi(0,1/5)+120\psi(0,1/5)^2-120\psi(1,6/5)$$
$$\left.+56\gamma\psi(1,1/5)+56\psi(0,1/5)\psi(1,1/5)-28\psi(2,6/5)\right) \tag{228}$$

$$\sum_{k=1}^{\infty}\frac{H(k)}{(3k+2)^2(5k+1)^3}=\frac{1}{48020}\left(108045+2700\gamma\psi(0,2/3)+1350\psi(0,2/3)^2-1350\psi(1,5/3)-420\gamma\psi(1,2/3)\right.$$
$$-420\psi(0,2/3)\psi(1,2/3)+210\psi(2,5/3)-2700\gamma\psi(0,1/5)-1350\psi(0,1/5)^2$$
$$+1350\psi(1,6/5)-840\gamma\psi(1,1/5)-840\psi(0,1/5)\psi(1,1/5)+420\psi(2,6/5)$$
$$\left.-98\gamma\psi(2,1/5)-98\psi(0,1/5)\psi(2,1/5)-98\psi(1,1/5)\psi(1,1/5)+49\psi(3,6/5)\right) \tag{229}$$

$$\sum_{k=1}^{\infty}\frac{H(k)}{(3k+2)^2(5k+2)}=\frac{-1}{96}\left(-33-30\gamma\psi(0,2/3)-15\psi(0,2/3)^2+15\psi(1,5/3)+8\gamma\psi(1,2/3)\right.$$
$$+8\psi(0,2/3)\psi(1,2/3)-4\psi(2,5/3)+30\gamma\psi(0,2/5)+15\psi(0,2/5)^2$$
$$\left.-15\psi(1,7/5)\right) \tag{230}$$

$$\sum_{k=1}^{\infty}\frac{H(k)}{(3k+2)^2(5k+2)^2}=\frac{-1}{64}\left(-16+30\gamma\psi(0,2/3)+15\psi(0,2/3)^2-15\psi(1,5/3)-4\gamma\psi(1,2/3)\right.$$
$$-4\psi(0,2/3)\psi(1,2/3)+2\psi(2,5/3)-30\gamma\psi(0,2/5)-15\psi(0,2/5)^2+15\psi(1,7/5)$$
$$\left.-4\gamma\psi(1,2/5)-4\psi(0,2/5)\psi(1,2/5)+2\psi(2,7/5)\right) \tag{231}$$

$$\sum_{k=1}^{\infty}\frac{H(k)}{(3k+2)^2(5k+2)^3}=\frac{-1}{2560}\left(-420-1350\gamma\psi(0,2/3)-675\psi(0,2/3)^2+675\psi(1,5/3)+120\gamma\psi(1,2/3)\right.$$
$$+120\psi(0,2/3)\psi(1,2/3)-60\psi(2,5/3)+1350\gamma\psi(0,2/5)+675\psi(0,2/5)^2-675\psi(1,7/5)$$
$$+240\gamma\psi(1,2/5)+240\psi(0,2/5)\psi(1,2/5)-120\psi(2,7/5)+16\gamma\psi(2,2/5)$$
$$\left.+16\psi(0,2/5)\psi(2,2/5)+16\psi(1,2/5)\psi(1,2/5)-8\psi(3,7/5)\right) \tag{232}$$

$$\sum_{k=1}^{\infty}\frac{H(k)}{(3k+2)^2(5k+3)}=\frac{-1}{36}\left(-7-180\gamma\psi(0,2/3)-90\psi(0,2/3)^2+90\psi(1,5/3)+12\gamma\psi(1,2/3)\right.$$
$$+12\psi(0,2/3)\psi(1,2/3)-6\psi(2,5/3)+180\gamma\psi(0,3/5)+90\psi(0,3/5)^2$$
$$\left.-90\psi(1,8/5)\right) \tag{233}$$

$$\sum_{k=1}^{\infty}\frac{H(k)}{(3k+2)^2(5k+3)^2}=\frac{1}{216}\big(19-6480\gamma\psi(0,2/3)-3240\psi(0,2/3)^2+3240\psi(1,5/3)+216\gamma\psi(1,2/3)$$
$$+216\psi(0,2/3)\psi(1,2/3)-108\psi(2,5/3)+6480\gamma\psi(0,3/5)+3240\psi(0,3/5)^2$$
$$-3240\psi(1,8/5)+216\gamma\psi(1,3/5)+216\psi(0,3/5)\psi(1,3/5)-108\psi(2,8/5)) \tag{234}$$

$$\sum_{k=1}^{\infty}\frac{H(k)}{(3k+2)^2(5k+3)^3}=\frac{1}{540}\big(20+72900\gamma\psi(0,2/3)+36450\psi(0,2/3)^2-36450\psi(1,5/3)-1620\gamma\psi(1,2/3)$$
$$-1620\psi(0,2/3)\psi(1,2/3)+810\psi(2,5/3)-72900\gamma\psi(0,3/5)-36450\psi(0,3/5)^2$$
$$+36450\psi(1,8/5)-3240\gamma\psi(1,3/5)-3240\psi(0,3/5)\psi(1,3/5)+1620\psi(2,8/5)$$
$$-54\gamma\psi(2,3/5)-54\psi(0,3/5)\psi(2,3/5)-54\psi(1,3/5)\psi(1,3/5)+27\psi(3,8/5)) \tag{235}$$

$$\sum_{k=1}^{\infty}\frac{H(k)}{(3k+2)^2(5k+4)}=\frac{1}{384}\big(51+480\gamma\psi(0,2/3)+240\psi(0,2/3)^2-240\psi(1,5/3)+64\gamma\psi(1,2/3)$$
$$+64\psi(0,2/3)\psi(1,2/3)-32\psi(2,5/3)-480\gamma\psi(0,4/5)-240\psi(0,4/5)^2$$
$$+240\psi(1,9/5)) \tag{236}$$

$$\sum_{k=1}^{\infty}\frac{H(k)}{(3k+2)^2(5k+4)^2}=\frac{-1}{256}\big(-11-960\gamma\psi(0,2/3)-480\psi(0,2/3)^2+480\psi(1,5/3)-64\gamma\psi(1,2/3)$$
$$-64\psi(0,2/3)\psi(1,2/3)+32\psi(2,5/3)+960\gamma\psi(0,4/5)+480\psi(0,4/5)^2-480\psi(1,9/5)$$
$$-64\gamma\psi(1,4/5)-64\psi(0,4/5)\psi(1,4/5)+32\psi(2,9/5)) \tag{237}$$

$$\sum_{k=1}^{\infty}\frac{H(k)}{(3k+2)^2(5k+4)^3}=\frac{-1}{10240}\big(-135-86400\gamma\psi(0,2/3)-43200\psi(0,2/3)^2+43200\psi(1,5/3)-3840\gamma\psi(1,2/3)$$
$$-3840\psi(0,2/3)\psi(1,2/3)+1920\psi(2,5/3)+86400\gamma\psi(0,4/5)+43200\psi(0,4/5)^2$$
$$-43200\psi(1,9/5)-7680\gamma\psi(1,4/5)-7680\psi(0,4/5)\psi(1,4/5)+3840\psi(2,9/5)$$
$$+256\gamma\psi(2,4/5)+256\psi(0,4/5)\psi(2,4/5)+256\psi(1,4/5)\psi(1,4/5)$$
$$-128\psi(3,9/5)) \tag{238}$$

$$\sum_{k=1}^{\infty}\frac{H(k)}{(3k+2)^3(4k+1)}=\frac{-1}{36000}\big(-19125-4608\gamma\psi(0,2/3)-2304\psi(0,2/3)^2+2304\psi(1,5/3)+1920\gamma\psi(1,2/3)$$
$$+1920\psi(0,2/3)\psi(1,2/3)-960\psi(2,5/3)-400\gamma\psi(2,2/3)-400\psi(0,2/3)\psi(2,2/3)$$
$$-400\psi(1,2/3)\psi(1,2/3)+200\psi(3,5/3)+4608\gamma\psi(0,1/4)+2304\psi(0,1/4)^2$$
$$-2304\psi(1,5/4)) \tag{239}$$

$$\sum_{k=1}^{\infty}\frac{H(k)}{(3k+2)^3(4k+1)^2}=\frac{-1}{60000}\big(-46875+13824\gamma\psi(0,2/3)+6912\psi(0,2/3)^2-6912\psi(1,5/3)-3840\gamma\psi(1,2/3)$$
$$-3840\psi(0,2/3)\psi(1,2/3)+1920\psi(2,5/3)+400\gamma\psi(2,2/3)+400\psi(0,2/3)\psi(2,2/3)$$
$$+400\psi(1,2/3)\psi(1,2/3)-200\psi(3,5/3)-13824\gamma\psi(0,1/4)-6912\psi(0,1/4)^2$$
$$+6912\psi(1,5/4)-1920\gamma\psi(1,1/4)-1920\psi(0,1/4)\psi(1,1/4)+960\psi(2,5/4)) \tag{240}$$

$$\sum_{k=1}^{\infty}\frac{H(k)}{(3k+2)^3(4k+1)^3}=\frac{1}{100000}\left(103125+27648\gamma\psi(0,2/3)+13824\psi(0,2/3)^2-13824\psi(1,5/3)-5760\gamma\psi(1,2/3)\right.$$
$$-5760\psi(0,2/3)\psi(1,2/3)+2880\psi(2,5/3)+400\gamma\psi(2,2/3)+400\psi(0,2/3)\psi(2,2/3)$$
$$+400\psi(1,2/3)\psi(1,2/3)-200\psi(3,5/3)-27648\gamma\psi(0,1/4)-13824\psi(0,1/4)^2$$
$$+13824\psi(1,5/4)-5760\gamma\psi(1,1/4)-5760\psi(0,1/4)\psi(1,1/4)+2880\psi(2,5/4)$$
$$-400\gamma\psi(2,1/4)-400\psi(0,1/4)\psi(2,1/4)-400\psi(1,1/4)\psi(1,1/4)$$
$$\left.+200\psi(3,5/4)\right) \tag{241}$$

$$\sum_{k=1}^{\infty}\frac{H(k)}{(3k+2)^3(4k+3)}=\frac{-1}{288}\left(-35+4608\gamma\psi(0,2/3)+2304\psi(0,2/3)^2-2304\psi(1,5/3)+384\gamma\psi(1,2/3)\right.$$
$$+384\psi(0,2/3)\psi(1,2/3)-192\psi(2,5/3)+16\gamma\psi(2,2/3)+16\psi(0,2/3)\psi(2,2/3)$$
$$+16\psi(1,2/3)\psi(1,2/3)-8\psi(3,5/3)-4608\gamma\psi(0,3/4)-2304\psi(0,3/4)^2$$
$$\left.+2304\psi(1,7/4)\right) \tag{242}$$

$$\sum_{k=1}^{\infty}\frac{H(k)}{(3k+2)^3(4k+3)^2}=\frac{1}{864}\left(43-124416\gamma\psi(0,2/3)-62208\psi(0,2/3)^2+62208\psi(1,5/3)-6912\gamma\psi(1,2/3)\right.$$
$$-6912\psi(0,2/3)\psi(1,2/3)+3456\psi(2,5/3)-144\gamma\psi(2,2/3)-144\psi(0,2/3)\psi(2,2/3)$$
$$-144\psi(1,2/3)\psi(1,2/3)+72\psi(3,5/3)+124416\gamma\psi(0,3/4)+62208\psi(0,3/4)^2$$
$$\left.-62208\psi(1,7/4)-3456\gamma\psi(1,3/4)-3456\psi(0,3/4)\psi(1,3/4)+1728\psi(2,7/4)\right) \tag{243}$$

$$\sum_{k=1}^{\infty}\frac{H(k)}{(3k+2)^3(4k+3)^3}=\frac{1}{864}\left(17-746496\gamma\psi(0,2/3)-373248\psi(0,2/3)^2+373248\psi(1,5/3)-31104\gamma\psi(1,2/3)\right.$$
$$-31104\psi(0,2/3)\psi(1,2/3)+15552\psi(2,5/3)-432\gamma\psi(2,2/3)-432\psi(0,2/3)\psi(2,2/3)$$
$$-432\psi(1,2/3)\psi(1,2/3)+216\psi(3,5/3)+746496\gamma\psi(0,3/4)+373248\psi(0,3/4)^2$$
$$-373248\psi(1,7/4)-31104\gamma\psi(1,3/4)-31104\psi(0,3/4)\psi(1,3/4)+15552\psi(2,7/4)$$
$$+432\gamma\psi(2,3/4)+432\psi(0,3/4)\psi(2,3/4)+432\psi(1,3/4)\psi(1,3/4)$$
$$\left.-216\psi(3,7/4)\right) \tag{244}$$

$$\sum_{k=1}^{\infty}\frac{H(k)}{(3k+2)^3(5k+1)}=\frac{1}{98784}\left(58653+7200\gamma\psi(0,2/3)+3600\psi(0,2/3)^2-3600\psi(1,5/3)-3360\gamma\psi(1,2/3)\right.$$
$$-3360\psi(0,2/3)\psi(1,2/3)+1680\psi(2,5/3)+784\gamma\psi(2,2/3)+784\psi(0,2/3)\psi(2,2/3)$$
$$+784\psi(1,2/3)\psi(1,2/3)-392\psi(3,5/3)-7200\gamma\psi(0,1/5)-3600\psi(0,1/5)^2$$
$$\left.+3600\psi(1,6/5)\right) \tag{245}$$

$$\sum_{k=1}^{\infty}\frac{H(k)}{(3k+2)^3(5k+1)^2}=\frac{1}{230496}\left(208887-21600\gamma\psi(0,2/3)-10800\psi(0,2/3)^2+10800\psi(1,5/3)+6720\gamma\psi(1,2/3)\right.$$
$$+6720\psi(0,2/3)\psi(1,2/3)-3360\psi(2,5/3)-784\gamma\psi(2,2/3)-784\psi(0,2/3)\psi(2,2/3)$$
$$-784\psi(1,2/3)\psi(1,2/3)+392\psi(3,5/3)+21600\gamma\psi(0,1/5)+10800\psi(0,1/5)^2$$
$$\left.-10800\psi(1,6/5)+3360\gamma\psi(1,1/5)+3360\psi(0,1/5)\psi(1,1/5)-1680\psi(2,6/5)\right) \tag{246}$$

$$\sum_{k=1}^{\infty}\frac{H(k)}{(3k+2)^3(5k+1)^3}=\frac{-1}{537824}\left(-655473-43200\gamma\psi(0,2/3)-21600\psi(0,2/3)^2+21600\psi(1,5/3)+10080\gamma\psi(1,2/3)\right.$$
$$+10080\psi(0,2/3)\psi(1,2/3)-5040\psi(2,5/3)-784\gamma\psi(2,2/3)-784\psi(0,2/3)\psi(2,2/3)$$
$$-784\psi(1,2/3)\psi(1,2/3)+392\psi(3,5/3)+43200\gamma\psi(0,1/5)+21600\psi(0,1/5)^2$$
$$-21600\psi(1,6/5)+10080\gamma\psi(1,1/5)+10080\psi(0,1/5)\psi(1,1/5)-5040\psi(2,6/5)$$
$$+784\gamma\psi(2,1/5)+784\psi(0,1/5)\psi(2,1/5)+784\psi(1,1/5)\psi(1,1/5)$$
$$\left.-392\psi(3,6/5)\right) \tag{247}$$

$$\sum_{k=1}^{\infty}\frac{H(k)}{(3k+2)^3(5k+2)}=\frac{1}{1152}\left(252+450\gamma\psi(0,2/3)+225\psi(0,2/3)^2-225\psi(1,5/3)-120\gamma\psi(1,2/3)\right.$$
$$-120\psi(0,2/3)\psi(1,2/3)+60\psi(2,5/3)+16\gamma\psi(2,2/3)+16\psi(0,2/3)\psi(2,2/3)$$
$$+16\psi(1,2/3)\psi(1,2/3)-8\psi(3,5/3)-450\gamma\psi(0,2/5)-225\psi(0,2/5)^2$$
$$\left.+225\psi(1,7/5)\right) \tag{248}$$

$$\sum_{k=1}^{\infty}\frac{H(k)}{(3k+2)^3(5k+2)^2}=\frac{-1}{1536}\left(-228+1350\gamma\psi(0,2/3)+675\psi(0,2/3)^2-675\psi(1,5/3)-240\gamma\psi(1,2/3)\right.$$
$$-240\psi(0,2/3)\psi(1,2/3)+120\psi(2,5/3)+16\gamma\psi(2,2/3)+16\psi(0,2/3)\psi(2,2/3)$$
$$+16\psi(1,2/3)\psi(1,2/3)-8\psi(3,5/3)-1350\gamma\psi(0,2/5)-675\psi(0,2/5)^2+675\psi(1,7/5)$$
$$\left.-120\gamma\psi(1,2/5)-120\psi(0,2/5)\psi(1,2/5)+60\psi(2,7/5)\right) \tag{249}$$

$$\sum_{k=1}^{\infty}\frac{H(k)}{(3k+2)^3(5k+2)^3}=\frac{1}{1024}\left(96+1350\gamma\psi(0,2/3)+675\psi(0,2/3)^2-675\psi(1,5/3)-180\gamma\psi(1,2/3)\right.$$
$$-180\psi(0,2/3)\psi(1,2/3)+90\psi(2,5/3)+8\gamma\psi(2,2/3)+8\psi(0,2/3)\psi(2,2/3)$$
$$+8\psi(1,2/3)\psi(1,2/3)-4\psi(3,5/3)-1350\gamma\psi(0,2/5)-675\psi(0,2/5)^2+675\psi(1,7/5)$$
$$-180\gamma\psi(1,2/5)-180\psi(0,2/5)\psi(1,2/5)+90\psi(2,7/5)-8\gamma\psi(2,2/5)$$
$$\left.-8\psi(0,2/5)\psi(2,2/5)-8\psi(1,2/5)\psi(1,2/5)+4\psi(3,7/5)\right) \tag{250}$$

$$\sum_{k=1}^{\infty}\frac{H(k)}{(3k+2)^3(5k+3)}=\frac{-1}{288}\left(-37-7200\gamma\psi(0,2/3)-3600\psi(0,2/3)^2+3600\psi(1,5/3)+480\gamma\psi(1,2/3)\right.$$
$$+480\psi(0,2/3)\psi(1,2/3)-240\psi(2,5/3)-16\gamma\psi(2,2/3)-16\psi(0,2/3)\psi(2,2/3)$$
$$-16\psi(1,2/3)\psi(1,2/3)+8\psi(3,5/3)+7200\gamma\psi(0,3/5)+3600\psi(0,3/5)^2$$
$$\left.-3600\psi(1,8/5)\right) \tag{251}$$

$$\sum_{k=1}^{\infty}\frac{H(k)}{(3k+2)^3(5k+3)^2}=\frac{1}{864}\left(47-194400\gamma\psi(0,2/3)-97200\psi(0,2/3)^2+97200\psi(1,5/3)+8640\gamma\psi(1,2/3)\right.$$
$$+8640\psi(0,2/3)\psi(1,2/3)-4320\psi(2,5/3)-144\gamma\psi(2,2/3)-144\psi(0,2/3)\psi(2,2/3)$$
$$-144\psi(1,2/3)\psi(1,2/3)+72\psi(3,5/3)+194400\gamma\psi(0,3/5)+97200\psi(0,3/5)^2$$
$$\left.-97200\psi(1,8/5)+4320\gamma\psi(1,3/5)+4320\psi(0,3/5)\psi(1,3/5)-2160\psi(2,8/5)\right) \tag{252}$$

$$\sum_{k=1}^{\infty}\frac{H(k)}{(3k+2)^3(5k+3)^3}=\frac{1}{864}\left(19+1166400\gamma\psi(0,2/3)+583200\psi(0,2/3)^2-583200\psi(1,5/3)-38880\gamma\psi(1,2/3)\right.$$
$$-38880\psi(0,2/3)\psi(1,2/3)+19440\psi(2,5/3)+432\gamma\psi(2,2/3)+432\psi(0,2/3)\psi(2,2/3)$$
$$+432\psi(1,2/3)\psi(1,2/3)-216\psi(3,5/3)-1166400\gamma\psi(0,3/5)-583200\psi(0,3/5)^2$$
$$+583200\psi(1,8/5)-38880\gamma\psi(1,3/5)-38880\psi(0,3/5)\psi(1,3/5)+19440\psi(2,8/5)$$
$$-432\gamma\psi(2,3/5)-432\psi(0,3/5)\psi(2,3/5)-432\psi(1,3/5)\psi(1,3/5)$$
$$\left.+216\psi(3,8/5)\right) \tag{253}$$

$$\sum_{k=1}^{\infty}\frac{H(k)}{(3k+2)^3(5k+4)}=\frac{-1}{2304}\left(-207+7200\gamma\psi(0,2/3)+3600\psi(0,2/3)^2-3600\psi(1,5/3)+960\gamma\psi(1,2/3)\right.$$
$$+960\psi(0,2/3)\psi(1,2/3)-480\psi(2,5/3)+64\gamma\psi(2,2/3)+64\psi(0,2/3)\psi(2,2/3)$$
$$+64\psi(1,2/3)\psi(1,2/3)-32\psi(3,5/3)-7200\gamma\psi(0,4/5)-3600\psi(0,4/5)^2$$
$$\left.+3600\psi(1,9/5)\right) \tag{254}$$

$$\sum_{k=1}^{\infty}\frac{H(k)}{(3k+2)^3(5k+4)^2}=\frac{-1}{768}\left(-21+10800\gamma\psi(0,2/3)+5400\psi(0,2/3)^2-5400\psi(1,5/3)+960\gamma\psi(1,2/3)\right.$$
$$+960\psi(0,2/3)\psi(1,2/3)-480\psi(2,5/3)+32\gamma\psi(2,2/3)+32\psi(0,2/3)\psi(2,2/3)$$
$$+32\psi(1,2/3)\psi(1,2/3)-16\psi(3,5/3)-10800\gamma\psi(0,4/5)-5400\psi(0,4/5)^2$$
$$\left.+5400\psi(1,9/5)+480\gamma\psi(1,4/5)+480\psi(0,4/5)\psi(1,4/5)-240\psi(2,9/5)\right) \tag{255}$$

$$\sum_{k=1}^{\infty}\frac{H(k)}{(3k+2)^3(5k+4)^3}=\frac{1}{4096}\left(33-172800\gamma\psi(0,2/3)-86400\psi(0,2/3)^2+86400\psi(1,5/3)-11520\gamma\psi(1,2/3)\right.$$
$$-11520\psi(0,2/3)\psi(1,2/3)+5760\psi(2,5/3)-256\gamma\psi(2,2/3)-256\psi(0,2/3)\psi(2,2/3)$$
$$-256\psi(1,2/3)\psi(1,2/3)+128\psi(3,5/3)+172800\gamma\psi(0,4/5)+86400\psi(0,4/5)^2$$
$$-86400\psi(1,9/5)-11520\gamma\psi(1,4/5)-11520\psi(0,4/5)\psi(1,4/5)+5760\psi(2,9/5)$$
$$+256\gamma\psi(2,4/5)+256\psi(0,4/5)\psi(2,4/5)+256\psi(1,4/5)\psi(1,4/5)$$
$$\left.-128\psi(3,9/5)\right) \tag{256}$$

$$\sum_{k=1}^{\infty}\frac{H(k)}{(4k+1)(5k+1)}=\frac{1}{2}\left(9+2\gamma\psi(0,1/4)+1\psi(0,1/4)^2-1\psi(1,5/4)-2\gamma\psi(0,1/5)-1\psi(0,1/5)^2\right.$$
$$\left.+1\psi(1,6/5)\right) \tag{257}$$

$$\sum_{k=1}^{\infty}\frac{H(k)}{(4k+1)(5k+1)^2}=\frac{1}{10}\left(70-40\gamma\psi(0,1/4)-20\psi(0,1/4)^2+20\psi(1,5/4)+40\gamma\psi(0,1/5)+20\psi(0,1/5)^2\right.$$
$$\left.-20\psi(1,6/5)+2\gamma\psi(1,1/5)+2\psi(0,1/5)\psi(1,1/5)-1\psi(2,6/5)\right) \tag{258}$$

$$\sum_{k=1}^{\infty}\frac{H(k)}{(4k+1)(5k+1)^3}=\frac{1}{100}\left(950+1600\gamma\psi(0,1/4)+800\psi(0,1/4)^2-800\psi(1,5/4)-1600\gamma\psi(0,1/5)\right.$$
$$-800\psi(0,1/5)^2+800\psi(1,6/5)-80\gamma\psi(1,1/5)-80\psi(0,1/5)\psi(1,1/5)+40\psi(2,6/5)$$
$$\left.-2\gamma\psi(2,1/5)-2\psi(0,1/5)\psi(2,1/5)-2\psi(1,1/5)\psi(1,1/5)+1\psi(3,6/5)\right) \tag{259}$$

$$\sum_{k=1}^{\infty}\frac{H(k)}{(4k+1)(5k+2)} = \frac{-1}{24}\left(-39+8\gamma\psi(0,1/4)+4\psi(0,1/4)^2-4\psi(1,5/4)-8\gamma\psi(0,2/5)-4\psi(0,2/5)^2\right.$$
$$\left.+4\psi(1,7/5)\right) \tag{260}$$

$$\sum_{k=1}^{\infty}\frac{H(k)}{(4k+1)(5k+2)^2} = \frac{1}{360}\left(405-160\gamma\psi(0,1/4)-80\psi(0,1/4)^2+80\psi(1,5/4)+160\gamma\psi(0,2/5)+80\psi(0,2/5)^2\right.$$
$$\left.-80\psi(1,7/5)-24\gamma\psi(1,2/5)-24\psi(0,2/5)\psi(1,2/5)+12\psi(2,7/5)\right) \tag{261}$$

$$\sum_{k=1}^{\infty}\frac{H(k)}{(4k+1)(5k+2)^3} = \frac{-1}{21600}\left(-15525+12800\gamma\psi(0,1/4)+6400\psi(0,1/4)^2-6400\psi(1,5/4)-12800\gamma\psi(0,2/5)\right.$$
$$-6400\psi(0,2/5)^2+6400\psi(1,7/5)+1920\gamma\psi(1,2/5)+1920\psi(0,2/5)\psi(1,2/5)$$
$$-960\psi(2,7/5)-144\gamma\psi(2,2/5)-144\psi(0,2/5)\psi(2,2/5)-144\psi(1,2/5)\psi(1,2/5)$$
$$\left.+72\psi(3,7/5)\right) \tag{262}$$

$$\sum_{k=1}^{\infty}\frac{H(k)}{(4k+1)(5k+3)} = \frac{1}{126}\left(119-18\gamma\psi(0,1/4)-9\psi(0,1/4)^2+9\psi(1,5/4)+18\gamma\psi(0,3/5)+9\psi(0,3/5)^2\right.$$
$$\left.-9\psi(1,8/5)\right) \tag{263}$$

$$\sum_{k=1}^{\infty}\frac{H(k)}{(4k+1)(5k+3)^2} = \frac{1}{13230}\left(5390-1080\gamma\psi(0,1/4)-540\psi(0,1/4)^2+540\psi(1,5/4)+1080\gamma\psi(0,3/5)\right.$$
$$+540\psi(0,3/5)^2-540\psi(1,8/5)-378\gamma\psi(1,3/5)-378\psi(0,3/5)\psi(1,3/5)$$
$$\left.+189\psi(2,8/5)\right) \tag{264}$$

$$\sum_{k=1}^{\infty}\frac{H(k)}{(4k+1)(5k+3)^3} = \frac{1}{102900}\left(17150-4800\gamma\psi(0,1/4)-2400\psi(0,1/4)^2+2400\psi(1,5/4)+4800\gamma\psi(0,3/5)\right.$$
$$+2400\psi(0,3/5)^2-2400\psi(1,8/5)-1680\gamma\psi(1,3/5)-1680\psi(0,3/5)\psi(1,3/5)$$
$$+840\psi(2,8/5)+294\gamma\psi(2,3/5)+294\psi(0,3/5)\psi(2,3/5)+294\psi(1,3/5)\psi(1,3/5)$$
$$\left.-147\psi(3,8/5)\right) \tag{265}$$

$$\sum_{k=1}^{\infty}\frac{H(k)}{(4k+1)(5k+4)} = \frac{1}{352}\left(231-32\gamma\psi(0,1/4)-16\psi(0,1/4)^2+16\psi(1,5/4)+32\gamma\psi(0,4/5)+16\psi(0,4/5)^2\right.$$
$$\left.-16\psi(1,9/5)\right) \tag{266}$$

$$\sum_{k=1}^{\infty}\frac{H(k)}{(4k+1)(5k+4)^2} = \frac{-1}{38720}\left(-7865+1280\gamma\psi(0,1/4)+640\psi(0,1/4)^2-640\psi(1,5/4)-1280\gamma\psi(0,4/5)\right.$$
$$-640\psi(0,4/5)^2+640\psi(1,9/5)+704\gamma\psi(1,4/5)+704\psi(0,4/5)\psi(1,4/5)$$
$$\left.-352\psi(2,9/5)\right) \tag{267}$$

$$\sum_{k=1}^{\infty}\frac{H(k)}{(4k+1)(5k+4)^3} = \frac{1}{17036800}\left(1031525-204800\gamma\psi(0,1/4)-102400\psi(0,1/4)^2+102400\psi(1,5/4)\right.$$
$$+204800\gamma\psi(0,4/5)+102400\psi(0,4/5)^2-102400\psi(1,9/5)-112640\gamma\psi(1,4/5)$$
$$-112640\psi(0,4/5)\psi(1,4/5)+56320\psi(2,9/5)+30976\gamma\psi(2,4/5)$$
$$\left.+30976\psi(0,4/5)\psi(2,4/5)+30976\psi(1,4/5)\psi(1,4/5)-15488\psi(3,9/5)\right) \tag{268}$$

$$\sum_{k=1}^{\infty}\frac{H(k)}{(4k+1)^2(5k+1)} = \frac{-1}{8}\big(-52-40\gamma\psi(0,1/4)-20\psi(0,1/4)^2+20\psi(1,5/4)+2\gamma\psi(1,1/4)$$
$$+2\psi(0,1/4)\psi(1,1/4)-1\psi(2,5/4)+40\gamma\psi(0,1/5)+20\psi(0,1/5)^2$$
$$-20\psi(1,6/5)) \tag{269}$$

$$\sum_{k=1}^{\infty}\frac{H(k)}{(4k+1)^2(5k+1)^2} = \frac{-1}{2}\big(-18+80\gamma\psi(0,1/4)+40\psi(0,1/4)^2-40\psi(1,5/4)-2\gamma\psi(1,1/4)$$
$$-2\psi(0,1/4)\psi(1,1/4)+1\psi(2,5/4)-80\gamma\psi(0,1/5)-40\psi(0,1/5)^2+40\psi(1,6/5)$$
$$-2\gamma\psi(1,1/5)-2\psi(0,1/5)\psi(1,1/5)+1\psi(2,6/5)) \tag{270}$$

$$\sum_{k=1}^{\infty}\frac{H(k)}{(4k+1)^2(5k+1)^3} = \frac{1}{20}\big(230+4800\gamma\psi(0,1/4)+2400\psi(0,1/4)^2-2400\psi(1,5/4)-80\gamma\psi(1,1/4)$$
$$-80\psi(0,1/4)\psi(1,1/4)+40\psi(2,5/4)-4800\gamma\psi(0,1/5)-2400\psi(0,1/5)^2$$
$$+2400\psi(1,6/5)-160\gamma\psi(1,1/5)-160\psi(0,1/5)\psi(1,1/5)+80\psi(2,6/5)$$
$$-2\gamma\psi(2,1/5)-2\psi(0,1/5)\psi(2,1/5)-2\psi(1,1/5)\psi(1,1/5)+1\psi(3,6/5)) \tag{271}$$

$$\sum_{k=1}^{\infty}\frac{H(k)}{(4k+1)^2(5k+2)} = \frac{1}{72}\big(189+40\gamma\psi(0,1/4)+20\psi(0,1/4)^2-20\psi(1,5/4)+6\gamma\psi(1,1/4)$$
$$+6\psi(0,1/4)\psi(1,1/4)-3\psi(2,5/4)-40\gamma\psi(0,2/5)-20\psi(0,2/5)^2$$
$$+20\psi(1,7/5)) \tag{272}$$

$$\sum_{k=1}^{\infty}\frac{H(k)}{(4k+1)^2(5k+2)^2} = \frac{1}{216}\big(351+320\gamma\psi(0,1/4)+160\psi(0,1/4)^2-160\psi(1,5/4)+24\gamma\psi(1,1/4)$$
$$+24\psi(0,1/4)\psi(1,1/4)-12\psi(2,5/4)-320\gamma\psi(0,2/5)-160\psi(0,2/5)^2+160\psi(1,7/5)$$
$$+24\gamma\psi(1,2/5)+24\psi(0,2/5)\psi(1,2/5)-12\psi(2,7/5)) \tag{273}$$

$$\sum_{k=1}^{\infty}\frac{H(k)}{(4k+1)^2(5k+2)^3} = \frac{-1}{4320}\big(-4185-12800\gamma\psi(0,1/4)-6400\psi(0,1/4)^2+6400\psi(1,5/4)-640\gamma\psi(1,1/4)$$
$$-640\psi(0,1/4)\psi(1,1/4)+320\psi(2,5/4)+12800\gamma\psi(0,2/5)+6400\psi(0,2/5)^2$$
$$-6400\psi(1,7/5)-1280\gamma\psi(1,2/5)-1280\psi(0,2/5)\psi(1,2/5)+640\psi(2,7/5)$$
$$+48\gamma\psi(2,2/5)+48\psi(0,2/5)\psi(2,2/5)+48\psi(1,2/5)\psi(1,2/5)-24\psi(3,7/5)) \tag{274}$$

$$\sum_{k=1}^{\infty}\frac{H(k)}{(4k+1)^2(5k+3)} = \frac{-1}{3528}\big(-5684-360\gamma\psi(0,1/4)-180\psi(0,1/4)^2+180\psi(1,5/4)-126\gamma\psi(1,1/4)$$
$$-126\psi(0,1/4)\psi(1,1/4)+63\psi(2,5/4)+360\gamma\psi(0,3/5)+180\psi(0,3/5)^2$$
$$-180\psi(1,8/5)) \tag{275}$$

$$\sum_{k=1}^{\infty}\frac{H(k)}{(4k+1)^2(5k+3)^2} = \frac{1}{18522}\big(11662+2160\gamma\psi(0,1/4)+1080\psi(0,1/4)^2-1080\psi(1,5/4)+378\gamma\psi(1,1/4)$$
$$+378\psi(0,1/4)\psi(1,1/4)-189\psi(2,5/4)-2160\gamma\psi(0,3/5)-1080\psi(0,3/5)^2$$
$$+1080\psi(1,8/5)+378\gamma\psi(1,3/5)+378\psi(0,3/5)\psi(1,3/5)-189\psi(2,8/5)) \tag{276}$$

$$\sum_{k=1}^{\infty}\frac{H(k)}{(4k+1)^2(5k+3)^3}=\frac{-1}{1296540}\big(-312130-129600\gamma\psi(0,1/4)-64800\psi(0,1/4)^2+64800\psi(1,5/4)$$
$$-15120\gamma\psi(1,1/4)-15120\psi(0,1/4)\psi(1,1/4)+7560\psi(2,5/4)+129600\gamma\psi(0,3/5)$$
$$+64800\psi(0,3/5)^2-64800\psi(1,8/5)-30240\gamma\psi(1,3/5)-30240\psi(0,3/5)\psi(1,3/5)$$
$$+15120\psi(2,8/5)+2646\gamma\psi(2,3/5)+2646\psi(0,3/5)\psi(2,3/5)+2646\psi(1,3/5)\psi(1,3/5)$$
$$-1323\psi(3,8/5)) \tag{277}$$

$$\sum_{k=1}^{\infty}\frac{H(k)}{(4k+1)^2(5k+4)}=\frac{-1}{3872}\big(-4477-160\gamma\psi(0,1/4)-80\psi(0,1/4)^2+80\psi(1,5/4)-88\gamma\psi(1,1/4)$$
$$-88\psi(0,1/4)\psi(1,1/4)+44\psi(2,5/4)+160\gamma\psi(0,4/5)+80\psi(0,4/5)^2$$
$$-80\psi(1,9/5)) \tag{278}$$

$$\sum_{k=1}^{\infty}\frac{H(k)}{(4k+1)^2(5k+4)^2}=\frac{-1}{85184}\big(-27951-2560\gamma\psi(0,1/4)-1280\psi(0,1/4)^2+1280\psi(1,5/4)-704\gamma\psi(1,1/4)$$
$$-704\psi(0,1/4)\psi(1,1/4)+352\psi(2,5/4)+2560\gamma\psi(0,4/5)+1280\psi(0,4/5)^2$$
$$-1280\psi(1,9/5)-704\gamma\psi(1,4/5)-704\psi(0,4/5)\psi(1,4/5)+352\psi(2,9/5)) \tag{279}$$

$$\sum_{k=1}^{\infty}\frac{H(k)}{(4k+1)^2(5k+4)^3}=\frac{-1}{37480960}\big(-3440635-614400\gamma\psi(0,1/4)-307200\psi(0,1/4)^2+307200\psi(1,5/4)$$
$$-112640\gamma\psi(1,1/4)-112640\psi(0,1/4)\psi(1,1/4)+56320\psi(2,5/4)+614400\gamma\psi(0,4/5)$$
$$+307200\psi(0,4/5)^2-307200\psi(1,9/5)-225280\gamma\psi(1,4/5)-225280\psi(0,4/5)\psi(1,4/5)$$
$$+112640\psi(2,9/5)+30976\gamma\psi(2,4/5)+30976\psi(0,4/5)\psi(2,4/5)$$
$$+30976\psi(1,4/5)\psi(1,4/5)-15488\psi(3,9/5)) \tag{280}$$

$$\sum_{k=1}^{\infty}\frac{H(k)}{(4k+1)^3(5k+1)}=\frac{-1}{64}\big(-544-1600\gamma\psi(0,1/4)-800\psi(0,1/4)^2+800\psi(1,5/4)+80\gamma\psi(1,1/4)$$
$$+80\psi(0,1/4)\psi(1,1/4)-40\psi(2,5/4)-2\gamma\psi(2,1/4)-2\psi(0,1/4)\psi(2,1/4)$$
$$-2\psi(1,1/4)\psi(1,1/4)+1\psi(3,5/4)+1600\gamma\psi(0,1/5)+800\psi(0,1/5)^2$$
$$-800\psi(1,6/5)) \tag{281}$$

$$\sum_{k=1}^{\infty}\frac{H(k)}{(4k+1)^3(5k+1)^2}=\frac{-1}{16}\big(-176+4800\gamma\psi(0,1/4)+2400\psi(0,1/4)^2-2400\psi(1,5/4)-160\gamma\psi(1,1/4)$$
$$-160\psi(0,1/4)\psi(1,1/4)+80\psi(2,5/4)+2\gamma\psi(2,1/4)+2\psi(0,1/4)\psi(2,1/4)$$
$$+2\psi(1,1/4)\psi(1,1/4)-1\psi(3,5/4)-4800\gamma\psi(0,1/5)-2400\psi(0,1/5)^2+2400\psi(1,6/5)$$
$$-80\gamma\psi(1,1/5)-80\psi(0,1/5)\psi(1,1/5)+40\psi(2,6/5)) \tag{282}$$

$$\sum_{k=1}^{\infty}\frac{H(k)}{(4k+1)^3(5k+1)^3}=\frac{-1}{4}\big(-54-9600\gamma\psi(0,1/4)-4800\psi(0,1/4)^2+4800\psi(1,5/4)+240\gamma\psi(1,1/4)$$
$$+240\psi(0,1/4)\psi(1,1/4)-120\psi(2,5/4)-2\gamma\psi(2,1/4)-2\psi(0,1/4)\psi(2,1/4)$$
$$-2\psi(1,1/4)\psi(1,1/4)+1\psi(3,5/4)+9600\gamma\psi(0,1/5)+4800\psi(0,1/5)^2-4800\psi(1,6/5)$$
$$+240\gamma\psi(1,1/5)+240\psi(0,1/5)\psi(1,1/5)-120\psi(2,6/5)+2\gamma\psi(2,1/5)$$
$$+2\psi(0,1/5)\psi(2,1/5)+2\psi(1,1/5)\psi(1,1/5)-1\psi(3,6/5)) \tag{283}$$

$$\sum_{k=1}^{\infty}\frac{H(k)}{(4k+1)^3(5k+2)} = \frac{1}{1728}\big(6264 - 1600\gamma\psi(0,1/4) - 800\psi(0,1/4)^2 + 800\psi(1,5/4) - 240\gamma\psi(1,1/4)$$
$$-240\psi(0,1/4)\psi(1,1/4) + 120\psi(2,5/4) - 18\gamma\psi(2,1/4) - 18\psi(0,1/4)\psi(2,1/4)$$
$$-18\psi(1,1/4)\psi(1,1/4) + 9\psi(3,5/4) + 1600\gamma\psi(0,2/5) + 800\psi(0,2/5)^2$$
$$-800\psi(1,7/5)) \tag{284}$$

$$\sum_{k=1}^{\infty}\frac{H(k)}{(4k+1)^3(5k+2)^2} = \frac{1}{432}\big(918 - 1600\gamma\psi(0,1/4) - 800\psi(0,1/4)^2 + 800\psi(1,5/4) - 160\gamma\psi(1,1/4)$$
$$-160\psi(0,1/4)\psi(1,1/4) + 80\psi(2,5/4) - 6\gamma\psi(2,1/4) - 6\psi(0,1/4)\psi(2,1/4)$$
$$-6\psi(1,1/4)\psi(1,1/4) + 3\psi(3,5/4) + 1600\gamma\psi(0,2/5) + 800\psi(0,2/5)^2 - 800\psi(1,7/5)$$
$$-80\gamma\psi(1,2/5) - 80\psi(0,2/5)\psi(1,2/5) + 40\psi(2,7/5)) \tag{285}$$

$$\sum_{k=1}^{\infty}\frac{H(k)}{(4k+1)^3(5k+2)^3} = \frac{1}{2592}\big(3159 - 25600\gamma\psi(0,1/4) - 12800\psi(0,1/4)^2 + 12800\psi(1,5/4) - 1920\gamma\psi(1,1/4)$$
$$-1920\psi(0,1/4)\psi(1,1/4) + 960\psi(2,5/4) - 48\gamma\psi(2,1/4) - 48\psi(0,1/4)\psi(2,1/4)$$
$$-48\psi(1,1/4)\psi(1,1/4) + 24\psi(3,5/4) + 25600\gamma\psi(0,2/5) + 12800\psi(0,2/5)^2$$
$$-12800\psi(1,7/5) - 1920\gamma\psi(1,2/5) - 1920\psi(0,2/5)\psi(1,2/5) + 960\psi(2,7/5)$$
$$+48\gamma\psi(2,2/5) + 48\psi(0,2/5)\psi(2,2/5) + 48\psi(1,2/5)\psi(1,2/5) - 24\psi(3,7/5)) \tag{286}$$

$$\sum_{k=1}^{\infty}\frac{H(k)}{(4k+1)^3(5k+3)} = \frac{-1}{197568}\big(-450016 + 14400\gamma\psi(0,1/4) + 7200\psi(0,1/4)^2 - 7200\psi(1,5/4) + 5040\gamma\psi(1,1/4)$$
$$+5040\psi(0,1/4)\psi(1,1/4) - 2520\psi(2,5/4) + 882\gamma\psi(2,1/4) + 882\psi(0,1/4)\psi(2,1/4)$$
$$+882\psi(1,1/4)\psi(1,1/4) - 441\psi(3,5/4) - 14400\gamma\psi(0,3/5) - 7200\psi(0,3/5)^2$$
$$+7200\psi(1,8/5)) \tag{287}$$

$$\sum_{k=1}^{\infty}\frac{H(k)}{(4k+1)^3(5k+3)^2} = \frac{-1}{1037232}\big(-883568 + 129600\gamma\psi(0,1/4) + 64800\psi(0,1/4)^2 - 64800\psi(1,5/4)$$
$$+30240\gamma\psi(1,1/4) + 30240\psi(0,1/4)\psi(1,1/4) - 15120\psi(2,5/4) + 2646\gamma\psi(2,1/4)$$
$$+2646\psi(0,1/4)\psi(2,1/4) + 2646\psi(1,1/4)\psi(1,1/4) - 1323\psi(3,5/4) - 129600\gamma\psi(0,3/5)$$
$$-64800\psi(0,3/5)^2 + 64800\psi(1,8/5) + 15120\gamma\psi(1,3/5) + 15120\psi(0,3/5)\psi(1,3/5)$$
$$-7560\psi(2,8/5)) \tag{288}$$

$$\sum_{k=1}^{\infty}\frac{H(k)}{(4k+1)^3(5k+3)^3} = \frac{-1}{1815156}\big(-571438 + 259200\gamma\psi(0,1/4) + 129600\psi(0,1/4)^2 - 129600\psi(1,5/4)$$
$$+45360\gamma\psi(1,1/4) + 45360\psi(0,1/4)\psi(1,1/4) - 22680\psi(2,5/4) + 2646\gamma\psi(2,1/4)$$
$$+2646\psi(0,1/4)\psi(2,1/4) + 2646\psi(1,1/4)\psi(1,1/4) - 1323\psi(3,5/4) - 259200\gamma\psi(0,3/5)$$
$$-129600\psi(0,3/5)^2 + 129600\psi(1,8/5) + 45360\gamma\psi(1,3/5) + 45360\psi(0,3/5)\psi(1,3/5)$$
$$-22680\psi(2,8/5) - 2646\gamma\psi(2,3/5) - 2646\psi(0,3/5)\psi(2,3/5) - 2646\psi(1,3/5)\psi(1,3/5)$$
$$+1323\psi(3,8/5)) \tag{289}$$

$$\sum_{k=1}^{\infty}\frac{H(k)}{(4k+1)^3(5k+4)} = \frac{-1}{85184}\left(-141086+1600\gamma\psi(0,1/4)+800\psi(0,1/4)^2-800\psi(1,5/4)+880\gamma\psi(1,1/4)\right.$$
$$+880\psi(0,1/4)\psi(1,1/4)-440\psi(2,5/4)+242\gamma\psi(2,1/4)+242\psi(0,1/4)\psi(2,1/4)$$
$$+242\psi(1,1/4)\psi(1,1/4)-121\psi(3,5/4)-1600\gamma\psi(0,4/5)-800\psi(0,4/5)^2$$
$$\left.+800\psi(1,9/5)\right) \tag{290}$$

$$\sum_{k=1}^{\infty}\frac{H(k)}{(4k+1)^3(5k+4)^2} = \frac{1}{937024}\left(424589-19200\gamma\psi(0,1/4)-9600\psi(0,1/4)^2+9600\psi(1,5/4)-7040\gamma\psi(1,1/4)\right.$$
$$-7040\psi(0,1/4)\psi(1,1/4)+3520\psi(2,5/4)-968\gamma\psi(2,1/4)-968\psi(0,1/4)\psi(2,1/4)$$
$$-968\psi(1,1/4)\psi(1,1/4)+484\psi(3,5/4)+19200\gamma\psi(0,4/5)+9600\psi(0,4/5)^2$$
$$\left.-9600\psi(1,9/5)-3520\gamma\psi(1,4/5)-3520\psi(0,4/5)\psi(1,4/5)+1760\psi(2,9/5)\right) \tag{291}$$

$$\sum_{k=1}^{\infty}\frac{H(k)}{(4k+1)^3(5k+4)^3} = \frac{-1}{82458112}\left(-10146213+1228800\gamma\psi(0,1/4)+614400\psi(0,1/4)^2-614400\psi(1,5/4)\right.$$
$$+337920\gamma\psi(1,1/4)+337920\psi(0,1/4)\psi(1,1/4)-168960\psi(2,5/4)+30976\gamma\psi(2,1/4)$$
$$+30976\psi(0,1/4)\psi(2,1/4)+30976\psi(1,1/4)\psi(1,1/4)-15488\psi(3,5/4)$$
$$-1228800\gamma\psi(0,4/5)-614400\psi(0,4/5)^2+614400\psi(1,9/5)+337920\gamma\psi(1,4/5)$$
$$+337920\psi(0,4/5)\psi(1,4/5)-168960\psi(2,9/5)-30976\gamma\psi(2,4/5)$$
$$\left.-30976\psi(0,4/5)\psi(2,4/5)-30976\psi(1,4/5)\psi(1,4/5)+15488\psi(3,9/5)\right) \tag{292}$$

$$\sum_{k=1}^{\infty}\frac{H(k)}{(4k+3)(5k+1)} = \frac{1}{198}\left(209+18\gamma\psi(0,3/4)+9\psi(0,3/4)^2-9\psi(1,7/4)-18\gamma\psi(0,1/5)-9\psi(0,1/5)^2\right.$$
$$\left.+9\psi(1,6/5)\right) \tag{293}$$

$$\sum_{k=1}^{\infty}\frac{H(k)}{(4k+3)(5k+1)^2} = \frac{1}{10890}\left(20570-360\gamma\psi(0,3/4)-180\psi(0,3/4)^2+180\psi(1,7/4)+360\gamma\psi(0,1/5)\right.$$
$$+180\psi(0,1/5)^2-180\psi(1,6/5)+198\gamma\psi(1,1/5)+198\psi(0,1/5)\psi(1,1/5)$$
$$\left.-99\psi(2,6/5)\right) \tag{294}$$

$$\sum_{k=1}^{\infty}\frac{H(k)}{(4k+3)(5k+1)^3} = \frac{1}{1197900}\left(3260950+14400\gamma\psi(0,3/4)+7200\psi(0,3/4)^2-7200\psi(1,7/4)-14400\gamma\psi(0,1/5)\right.$$
$$-7200\psi(0,1/5)^2+7200\psi(1,6/5)-7920\gamma\psi(1,1/5)-7920\psi(0,1/5)\psi(1,1/5)$$
$$+3960\psi(2,6/5)-2178\gamma\psi(2,1/5)-2178\psi(0,1/5)\psi(2,1/5)-2178\psi(1,1/5)\psi(1,1/5)$$
$$\left.+1089\psi(3,6/5)\right) \tag{295}$$

$$\sum_{k=1}^{\infty}\frac{H(k)}{(4k+3)(5k+2)} = \frac{1}{504}\left(161+72\gamma\psi(0,3/4)+36\psi(0,3/4)^2-36\psi(1,7/4)-72\gamma\psi(0,2/5)-36\psi(0,2/5)^2\right.$$
$$\left.+36\psi(1,7/5)\right) \tag{296}$$

$$\sum_{k=1}^{\infty}\frac{H(k)}{(4k+3)(5k+2)^2}=\frac{-1}{17640}\left(-4655+1440\gamma\psi(0,3/4)+720\psi(0,3/4)^2-720\psi(1,7/4)-1440\gamma\psi(0,2/5)\right.$$
$$-720\psi(0,2/5)^2+720\psi(1,7/5)-504\gamma\psi(1,2/5)-504\psi(0,2/5)\psi(1,2/5)$$
$$\left.+252\psi(2,7/5)\right) \tag{297}$$

$$\sum_{k=1}^{\infty}\frac{H(k)}{(4k+3)(5k+2)^3}=\frac{1}{2469600}\left(454475+115200\gamma\psi(0,3/4)+57600\psi(0,3/4)^2-57600\psi(1,7/4)-115200\gamma\psi(0,2/5)\right.$$
$$-57600\psi(0,2/5)^2+57600\psi(1,7/5)-40320\gamma\psi(1,2/5)-40320\psi(0,2/5)\psi(1,2/5)$$
$$+20160\psi(2,7/5)-7056\gamma\psi(2,2/5)-7056\psi(0,2/5)\psi(2,2/5)-7056\psi(1,2/5)\psi(1,2/5)$$
$$\left.+3528\psi(3,7/5)\right) \tag{298}$$

$$\sum_{k=1}^{\infty}\frac{H(k)}{(4k+3)(5k+3)}=\frac{1}{6}\left(1+2\gamma\psi(0,3/4)+1\psi(0,3/4)^2-1\psi(1,7/4)-2\gamma\psi(0,3/5)-1\psi(0,3/5)^2\right.$$
$$\left.+1\psi(1,8/5)\right) \tag{299}$$

$$\sum_{k=1}^{\infty}\frac{H(k)}{(4k+3)(5k+3)^2}=\frac{1}{810}\left(70-360\gamma\psi(0,3/4)-180\psi(0,3/4)^2+180\psi(1,7/4)+360\gamma\psi(0,3/5)+180\psi(0,3/5)^2\right.$$
$$\left.-180\psi(1,8/5)+54\gamma\psi(1,3/5)+54\psi(0,3/5)\psi(1,3/5)-27\psi(2,8/5)\right) \tag{300}$$

$$\sum_{k=1}^{\infty}\frac{H(k)}{(4k+3)(5k+3)^3}=\frac{-1}{24300}\left(-950-14400\gamma\psi(0,3/4)-7200\psi(0,3/4)^2+7200\psi(1,7/4)+14400\gamma\psi(0,3/5)\right.$$
$$+7200\psi(0,3/5)^2-7200\psi(1,8/5)+2160\gamma\psi(1,3/5)+2160\psi(0,3/5)\psi(1,3/5)$$
$$-1080\psi(2,8/5)+162\gamma\psi(2,3/5)+162\psi(0,3/5)\psi(2,3/5)+162\psi(1,3/5)\psi(1,3/5)$$
$$\left.-81\psi(3,8/5)\right) \tag{301}$$

$$\sum_{k=1}^{\infty}\frac{H(k)}{(4k+3)(5k+4)}=\frac{1}{288}\left(31-288\gamma\psi(0,3/4)-144\psi(0,3/4)^2+144\psi(1,7/4)+288\gamma\psi(0,4/5)+144\psi(0,4/5)^2\right.$$
$$\left.-144\psi(1,9/5)\right) \tag{302}$$

$$\sum_{k=1}^{\infty}\frac{H(k)}{(4k+3)(5k+4)^2}=\frac{-1}{2880}\left(-115+11520\gamma\psi(0,3/4)+5760\psi(0,3/4)^2-5760\psi(1,7/4)-11520\gamma\psi(0,4/5)\right.$$
$$-5760\psi(0,4/5)^2+5760\psi(1,9/5)+576\gamma\psi(1,4/5)+576\psi(0,4/5)\psi(1,4/5)$$
$$\left.-288\psi(2,9/5)\right) \tag{303}$$

$$\sum_{k=1}^{\infty}\frac{H(k)}{(4k+3)(5k+4)^3}=\frac{1}{115200}\left(1525-1843200\gamma\psi(0,3/4)-921600\psi(0,3/4)^2+921600\psi(1,7/4)\right.$$
$$+1843200\gamma\psi(0,4/5)+921600\psi(0,4/5)^2-921600\psi(1,9/5)-92160\gamma\psi(1,4/5)$$
$$-92160\psi(0,4/5)\psi(1,4/5)+46080\psi(2,9/5)+2304\gamma\psi(2,4/5)+2304\psi(0,4/5)\psi(2,4/5)$$
$$\left.+2304\psi(1,4/5)\psi(1,4/5)-1152\psi(3,9/5)\right) \tag{304}$$

$$\sum_{k=1}^{\infty}\frac{H(k)}{(4k+3)^2(5k+1)}=\frac{-1}{26136}\left(-11132-1080\gamma\psi(0,3/4)-540\psi(0,3/4)^2+540\psi(1,7/4)+594\gamma\psi(1,3/4)\right.$$
$$+594\psi(0,3/4)\psi(1,3/4)-297\psi(2,7/4)+1080\gamma\psi(0,1/5)+540\psi(0,1/5)^2$$
$$\left.-540\psi(1,6/5)\right) \tag{305}$$

$$\sum_{k=1}^{\infty}\frac{H(k)}{(4k+3)^2(5k+1)^2}=\frac{1}{71874}\left(50578-2160\gamma\psi(0,3/4)-1080\psi(0,3/4)^2+1080\psi(1,7/4)+594\gamma\psi(1,3/4)\right.$$
$$+594\psi(0,3/4)\psi(1,3/4)-297\psi(2,7/4)+2160\gamma\psi(0,1/5)+1080\psi(0,1/5)^2$$
$$\left.-1080\psi(1,6/5)+594\gamma\psi(1,1/5)+594\psi(0,1/5)\psi(1,1/5)-297\psi(2,6/5)\right) \tag{306}$$

$$\sum_{k=1}^{\infty}\frac{H(k)}{(4k+3)^2(5k+1)^3}=\frac{1}{7906140}\left(7759730+129600\gamma\psi(0,3/4)+64800\psi(0,3/4)^2-64800\psi(1,7/4)\right.$$
$$-23760\gamma\psi(1,3/4)-23760\psi(0,3/4)\psi(1,3/4)+11880\psi(2,7/4)-129600\gamma\psi(0,1/5)$$
$$-64800\psi(0,1/5)^2+64800\psi(1,6/5)-47520\gamma\psi(1,1/5)-47520\psi(0,1/5)\psi(1,1/5)$$
$$+23760\psi(2,6/5)-6534\gamma\psi(2,1/5)-6534\psi(0,1/5)\psi(2,1/5)-6534\psi(1,1/5)\psi(1,1/5)$$
$$\left.+3267\psi(3,6/5)\right) \tag{307}$$

$$\sum_{k=1}^{\infty}\frac{H(k)}{(4k+3)^2(5k+2)}=\frac{-1}{10584}\left(-1519-1080\gamma\psi(0,3/4)-540\psi(0,3/4)^2+540\psi(1,7/4)+378\gamma\psi(1,3/4)\right.$$
$$+378\psi(0,3/4)\psi(1,3/4)-189\psi(2,7/4)+1080\gamma\psi(0,2/5)+540\psi(0,2/5)^2$$
$$\left.-540\psi(1,7/5)\right) \tag{308}$$

$$\sum_{k=1}^{\infty}\frac{H(k)}{(4k+3)^2(5k+2)^2}=\frac{1}{74088}\left(7889-8640\gamma\psi(0,3/4)-4320\psi(0,3/4)^2+4320\psi(1,7/4)+1512\gamma\psi(1,3/4)\right.$$
$$+1512\psi(0,3/4)\psi(1,3/4)-756\psi(2,7/4)+8640\gamma\psi(0,2/5)+4320\psi(0,2/5)^2$$
$$\left.-4320\psi(1,7/5)+1512\gamma\psi(1,2/5)+1512\psi(0,2/5)\psi(1,2/5)-756\psi(2,7/5)\right) \tag{309}$$

$$\sum_{k=1}^{\infty}\frac{H(k)}{(4k+3)^2(5k+2)^3}=\frac{1}{10372320}\left(732305+1036800\gamma\psi(0,3/4)+518400\psi(0,3/4)^2-518400\psi(1,7/4)\right.$$
$$-120960\gamma\psi(1,3/4)-120960\psi(0,3/4)\psi(1,3/4)+60480\psi(2,7/4)-1036800\gamma\psi(0,2/5)$$
$$-518400\psi(0,2/5)^2+518400\psi(1,7/5)-241920\gamma\psi(1,2/5)-241920\psi(0,2/5)\psi(1,2/5)$$
$$+120960\psi(2,7/5)-21168\gamma\psi(2,2/5)-21168\psi(0,2/5)\psi(2,2/5)$$
$$\left.-21168\psi(1,2/5)\psi(1,2/5)+10584\psi(3,7/5)\right) \tag{310}$$

$$\sum_{k=1}^{\infty}\frac{H(k)}{(4k+3)^2(5k+3)}=\frac{1}{648}\left(52+360\gamma\psi(0,3/4)+180\psi(0,3/4)^2-180\psi(1,7/4)-54\gamma\psi(1,3/4)\right.$$
$$-54\psi(0,3/4)\psi(1,3/4)+27\psi(2,7/4)-360\gamma\psi(0,3/5)-180\psi(0,3/5)^2$$
$$\left.+180\psi(1,8/5)\right) \tag{311}$$

$$\sum_{k=1}^{\infty}\frac{H(k)}{(4k+3)^2(5k+3)^2}=\frac{1}{54}\left(2-80\gamma\psi(0,3/4)-40\psi(0,3/4)^2+40\psi(1,7/4)+6\gamma\psi(1,3/4)\right.$$
$$+6\psi(0,3/4)\psi(1,3/4)-3\psi(2,7/4)+80\gamma\psi(0,3/5)+40\psi(0,3/5)^2-40\psi(1,8/5)$$
$$\left.+6\gamma\psi(1,3/5)+6\psi(0,3/5)\psi(1,3/5)-3\psi(2,8/5)\right) \tag{312}$$

$$\sum_{k=1}^{\infty}\frac{H(k)}{(4k+3)^2(5k+3)^3}=\frac{-1}{14580}\left(-230-43200\gamma\psi(0,3/4)-21600\psi(0,3/4)^2+21600\psi(1,7/4)+2160\gamma\psi(1,3/4)\right.$$
$$+2160\psi(0,3/4)\psi(1,3/4)-1080\psi(2,7/4)+43200\gamma\psi(0,3/5)+21600\psi(0,3/5)^2$$
$$-21600\psi(1,8/5)+4320\gamma\psi(1,3/5)+4320\psi(0,3/5)\psi(1,3/5)-2160\psi(2,8/5)$$
$$\left.+162\gamma\psi(2,3/5)+162\psi(0,3/5)\psi(2,3/5)+162\psi(1,3/5)\psi(1,3/5)-81\psi(3,8/5)\right) \tag{313}$$

$$\sum_{k=1}^{\infty}\frac{H(k)}{(4k+3)^2(5k+4)}=\frac{1}{864}\left(47+4320\gamma\psi(0,3/4)+2160\psi(0,3/4)^2-2160\psi(1,7/4)+216\gamma\psi(1,3/4)\right.$$
$$+216\psi(0,3/4)\psi(1,3/4)-108\psi(2,7/4)-4320\gamma\psi(0,4/5)-2160\psi(0,4/5)^2$$
$$\left.+2160\psi(1,9/5)\right) \tag{314}$$

$$\sum_{k=1}^{\infty}\frac{H(k)}{(4k+3)^2(5k+4)^2}=\frac{-1}{1728}\left(-31-69120\gamma\psi(0,3/4)-34560\psi(0,3/4)^2+34560\psi(1,7/4)-1728\gamma\psi(1,3/4)\right.$$
$$-1728\psi(0,3/4)\psi(1,3/4)+864\psi(2,7/4)+69120\gamma\psi(0,4/5)+34560\psi(0,4/5)^2$$
$$\left.-34560\psi(1,9/5)-1728\gamma\psi(1,4/5)-1728\psi(0,4/5)\psi(1,4/5)+864\psi(2,9/5)\right) \tag{315}$$

$$\sum_{k=1}^{\infty}\frac{H(k)}{(4k+3)^2(5k+4)^3}=\frac{1}{69120}\left(385+16588800\gamma\psi(0,3/4)+8294400\psi(0,3/4)^2-8294400\psi(1,7/4)\right.$$
$$+276480\gamma\psi(1,3/4)+276480\psi(0,3/4)\psi(1,3/4)-138240\psi(2,7/4)$$
$$-16588800\gamma\psi(0,4/5)-8294400\psi(0,4/5)^2+8294400\psi(1,9/5)+552960\gamma\psi(1,4/5)$$
$$+552960\psi(0,4/5)\psi(1,4/5)-276480\psi(2,9/5)-6912\gamma\psi(2,4/5)$$
$$\left.-6912\psi(0,4/5)\psi(2,4/5)-6912\psi(1,4/5)\psi(1,4/5)+3456\psi(3,9/5)\right) \tag{316}$$

$$\sum_{k=1}^{\infty}\frac{H(k)}{(4k+3)^3(5k+1)}=\frac{-1}{255552}\left(-42592-4800\gamma\psi(0,3/4)-2400\psi(0,3/4)^2+2400\psi(1,7/4)+2640\gamma\psi(1,3/4)\right.$$
$$+2640\psi(0,3/4)\psi(1,3/4)-1320\psi(2,7/4)-726\gamma\psi(2,3/4)-726\psi(0,3/4)\psi(2,3/4)$$
$$-726\psi(1,3/4)\psi(1,3/4)+363\psi(3,7/4)+4800\gamma\psi(0,1/5)+2400\psi(0,1/5)^2$$
$$\left.-2400\psi(1,6/5)\right) \tag{317}$$

$$\sum_{k=1}^{\infty}\frac{H(k)}{(4k+3)^3(5k+1)^2}=\frac{-1}{6324912}\left(-1639792+129600\gamma\psi(0,3/4)+64800\psi(0,3/4)^2-64800\psi(1,7/4)\right.$$
$$-47520\gamma\psi(1,3/4)-47520\psi(0,3/4)\psi(1,3/4)+23760\psi(2,7/4)+6534\gamma\psi(2,3/4)$$
$$+6534\psi(0,3/4)\psi(2,3/4)+6534\psi(1,3/4)\psi(1,3/4)-3267\psi(3,7/4)-129600\gamma\psi(0,1/5)$$
$$-64800\psi(0,1/5)^2+64800\psi(1,6/5)-23760\gamma\psi(1,1/5)-23760\psi(0,1/5)\psi(1,1/5)$$
$$\left.+11880\psi(2,6/5)\right) \tag{318}$$

$$\sum_{k=1}^{\infty}\frac{H(k)}{(4k+3)^3(5k+1)^3}=\frac{1}{17393508}\big(6119938+259200\gamma\psi(0,3/4)+129600\psi(0,3/4)^2-129600\psi(1,7/4)$$
$$-71280\gamma\psi(1,3/4)-71280\psi(0,3/4)\psi(1,3/4)+35640\psi(2,7/4)+6534\gamma\psi(2,3/4)$$
$$+6534\psi(0,3/4)\psi(2,3/4)+6534\psi(1,3/4)\psi(1,3/4)-3267\psi(3,7/4)-259200\gamma\psi(0,1/5)$$
$$-129600\psi(0,1/5)^2+129600\psi(1,6/5)-71280\gamma\psi(1,1/5)-71280\psi(0,1/5)\psi(1,1/5)$$
$$+35640\psi(2,6/5)-6534\gamma\psi(2,1/5)-6534\psi(0,1/5)\psi(2,1/5)-6534\psi(1,1/5)\psi(1,1/5)$$
$$+3267\psi(3,6/5)) \tag{319}$$

$$\sum_{k=1}^{\infty}\frac{H(k)}{(4k+3)^3(5k+2)}=\frac{1}{592704}\big(35672+43200\gamma\psi(0,3/4)+21600\psi(0,3/4)^2-21600\psi(1,7/4)-15120\gamma\psi(1,3/4)$$
$$-15120\psi(0,3/4)\psi(1,3/4)+7560\psi(2,7/4)+2646\gamma\psi(2,3/4)+2646\psi(0,3/4)\psi(2,3/4)$$
$$+2646\psi(1,3/4)\psi(1,3/4)-1323\psi(3,7/4)-43200\gamma\psi(0,2/5)-21600\psi(0,2/5)^2$$
$$+21600\psi(1,7/5)) \tag{320}$$

$$\sum_{k=1}^{\infty}\frac{H(k)}{(4k+3)^3(5k+2)^2}=\frac{-1}{115248}\big(-4802+14400\gamma\psi(0,3/4)+7200\psi(0,3/4)^2-7200\psi(1,7/4)-3360\gamma\psi(1,3/4)$$
$$-3360\psi(0,3/4)\psi(1,3/4)+1680\psi(2,7/4)+294\gamma\psi(2,3/4)+294\psi(0,3/4)\psi(2,3/4)$$
$$+294\psi(1,3/4)\psi(1,3/4)-147\psi(3,7/4)-14400\gamma\psi(0,2/5)-7200\psi(0,2/5)^2$$
$$+7200\psi(1,7/5)-1680\gamma\psi(1,2/5)-1680\psi(0,2/5)\psi(1,2/5)+840\psi(2,7/5)) \tag{321}$$

$$\sum_{k=1}^{\infty}\frac{H(k)}{(4k+3)^3(5k+2)^3}=\frac{1}{14521248}\big(386561+2073600\gamma\psi(0,3/4)+1036800\psi(0,3/4)^2-1036800\psi(1,7/4)$$
$$-362880\gamma\psi(1,3/4)-362880\psi(0,3/4)\psi(1,3/4)+181440\psi(2,7/4)+21168\gamma\psi(2,3/4)$$
$$+21168\psi(0,3/4)\psi(2,3/4)+21168\psi(1,3/4)\psi(1,3/4)-10584\psi(3,7/4)$$
$$-2073600\gamma\psi(0,2/5)-1036800\psi(0,2/5)^2+1036800\psi(1,7/5)-362880\gamma\psi(1,2/5)$$
$$-362880\psi(0,2/5)\psi(1,2/5)+181440\psi(2,7/5)-21168\gamma\psi(2,2/5)$$
$$-21168\psi(0,2/5)\psi(2,2/5)-21168\psi(1,2/5)\psi(1,2/5)+10584\psi(3,7/5)) \tag{322}$$

$$\sum_{k=1}^{\infty}\frac{H(k)}{(4k+3)^3(5k+3)}=\frac{-1}{15552}\big(-544-14400\gamma\psi(0,3/4)-7200\psi(0,3/4)^2+7200\psi(1,7/4)+2160\gamma\psi(1,3/4)$$
$$+2160\psi(0,3/4)\psi(1,3/4)-1080\psi(2,7/4)-162\gamma\psi(2,3/4)-162\psi(0,3/4)\psi(2,3/4)$$
$$-162\psi(1,3/4)\psi(1,3/4)+81\psi(3,7/4)+14400\gamma\psi(0,3/5)+7200\psi(0,3/5)^2$$
$$-7200\psi(1,8/5)) \tag{323}$$

$$\sum_{k=1}^{\infty}\frac{H(k)}{(4k+3)^3(5k+3)^2}=\frac{1}{11664}\big(176-43200\gamma\psi(0,3/4)-21600\psi(0,3/4)^2+21600\psi(1,7/4)+4320\gamma\psi(1,3/4)$$
$$+4320\psi(0,3/4)\psi(1,3/4)-2160\psi(2,7/4)-162\gamma\psi(2,3/4)-162\psi(0,3/4)\psi(2,3/4)$$
$$-162\psi(1,3/4)\psi(1,3/4)+81\psi(3,7/4)+43200\gamma\psi(0,3/5)+21600\psi(0,3/5)^2$$
$$-21600\psi(1,8/5)+2160\gamma\psi(1,3/5)+2160\psi(0,3/5)\psi(1,3/5)-1080\psi(2,8/5)) \tag{324}$$

$$\sum_{k=1}^{\infty}\frac{H(k)}{(4k+3)^3(5k+3)^3}=\frac{1}{324}\big(2+3200\gamma\psi(0,3/4)+1600\psi(0,3/4)^2-1600\psi(1,7/4)-240\gamma\psi(1,3/4)$$
$$-240\psi(0,3/4)\psi(1,3/4)+120\psi(2,7/4)+6\gamma\psi(2,3/4)+6\psi(0,3/4)\psi(2,3/4)$$
$$+6\psi(1,3/4)\psi(1,3/4)-3\psi(3,7/4)-3200\gamma\psi(0,3/5)-1600\psi(0,3/5)^2+1600\psi(1,8/5)$$
$$-240\gamma\psi(1,3/5)-240\psi(0,3/5)\psi(1,3/5)+120\psi(2,8/5)-6\gamma\psi(2,3/5)$$
$$-6\psi(0,3/5)\psi(2,3/5)-6\psi(1,3/5)\psi(1,3/5)+3\psi(3,8/5)) \tag{325}$$

$$\sum_{k=1}^{\infty}\frac{H(k)}{(4k+3)^3(5k+4)}=\frac{1}{576}\big(14-14400\gamma\psi(0,3/4)-7200\psi(0,3/4)^2+7200\psi(1,7/4)-720\gamma\psi(1,3/4)$$
$$-720\psi(0,3/4)\psi(1,3/4)+360\psi(2,7/4)-18\gamma\psi(2,3/4)-18\psi(0,3/4)\psi(2,3/4)$$
$$-18\psi(1,3/4)\psi(1,3/4)+9\psi(3,7/4)+14400\gamma\psi(0,4/5)+7200\psi(0,4/5)^2$$
$$-7200\psi(1,9/5)) \tag{326}$$

$$\sum_{k=1}^{\infty}\frac{H(k)}{(4k+3)^3(5k+4)^2}=\frac{1}{1728}\big(13-518400\gamma\psi(0,3/4)-259200\psi(0,3/4)^2+259200\psi(1,7/4)-17280\gamma\psi(1,3/4)$$
$$-17280\psi(0,3/4)\psi(1,3/4)+8640\psi(2,7/4)-216\gamma\psi(2,3/4)-216\psi(0,3/4)\psi(2,3/4)$$
$$-216\psi(1,3/4)\psi(1,3/4)+108\psi(3,7/4)+518400\gamma\psi(0,4/5)+259200\psi(0,4/5)^2$$
$$-259200\psi(1,9/5)-8640\gamma\psi(1,4/5)-8640\psi(0,4/5)\psi(1,4/5)+4320\psi(2,9/5)) \tag{327}$$

$$\sum_{k=1}^{\infty}\frac{H(k)}{(4k+3)^3(5k+4)^3}=\frac{-1}{13824}\big(-31+33177600\gamma\psi(0,3/4)+16588800\psi(0,3/4)^2-16588800\psi(1,7/4)$$
$$+829440\gamma\psi(1,3/4)+829440\psi(0,3/4)\psi(1,3/4)-414720\psi(2,7/4)+6912\gamma\psi(2,3/4)$$
$$+6912\psi(0,3/4)\psi(2,3/4)+6912\psi(1,3/4)\psi(1,3/4)-3456\psi(3,7/4)$$
$$-33177600\gamma\psi(0,4/5)-16588800\psi(0,4/5)^2+16588800\psi(1,9/5)+829440\gamma\psi(1,4/5)$$
$$+829440\psi(0,4/5)\psi(1,4/5)-414720\psi(2,9/5)-6912\gamma\psi(2,4/5)$$
$$-6912\psi(0,4/5)\psi(2,4/5)-6912\psi(1,4/5)\psi(1,4/5)+3456\psi(3,9/5)) \tag{328}$$

$$\sum_{k=1}^{\infty}\frac{H(k)}{(k^2+3k+1)} = \frac{1}{2}\left(1/q_1^2/q_d - 2\gamma\psi(0,q_1)/q_d - 1\psi(0,q_1)^2/q_d + 1\psi(1,1+q_1)/q_d - 1/q_2^2/q_d\right.$$
$$\left.+2\gamma\psi(0,q_2)/q_d + 1\psi(0,q_2)^2/q_d - 1\psi(1,1+q_2)/q_d\right) \tag{329}$$
$$q_d=\sqrt{5},\ q_1=(3-\sqrt{5})/2,\ q_2=(3+\sqrt{5})/2$$

$$\sum_{k=1}^{\infty}\frac{H(k)}{(k^2+3k+1)^2} = \frac{-1}{10}\left(400/q_1^2/q_d - 4\gamma\psi(0,q_1)/q_d - 2\psi(0,q_1)^2/q_d + 2\psi(1,1+q_1)/q_d - 70/q_1^3\right.$$
$$-2\gamma\psi(1,q_1) - 2\psi(0,q_1)\psi(1,q_1) + 1\psi(2,1+q_1) + 4\gamma\psi(0,q_2)/q_d$$
$$+2\psi(0,q_2)^2/q_d - 2\psi(1,1+q_2)/q_d - 2\gamma\psi(1,q_2) - 2\psi(0,q_2)\psi(1,q_2)$$
$$\left.+1\psi(2,1+q_2)\right) \tag{330}$$
$$q_d=\sqrt{5},\ q_1=(3-\sqrt{5})/2,\ q_2=(3+\sqrt{5})/2$$

$$\sum_{k=1}^{\infty}\frac{H(k)}{(k^2+4k+1)} = \frac{1}{2}\left(1/q_1^2/q_d - 2\gamma\psi(0,q_1)/q_d - 1\psi(0,q_1)^2/q_d + 1\psi(1,1+q_1)/q_d - 1/q_2^2/q_d\right.$$
$$\left.+2\gamma\psi(0,q_2)/q_d + 1\psi(0,q_2)^2/q_d - 1\psi(1,1+q_2)/q_d\right) \tag{331}$$
$$q_d=\sqrt{12},\ q_1=(4-\sqrt{12})/2,\ q_2=(4+\sqrt{12})/2$$

$$\sum_{k=1}^{\infty}\frac{H(k)}{(k^2+4k+1)^2} = \frac{1}{24}\left(-4320/q_1^2/q_d + 4\gamma\psi(0,q_1)/q_d + 2\psi(0,q_1)^2/q_d - 2\psi(1,1+q_1)/q_d + 336/q_1^3\right.$$
$$+2\gamma\psi(1,q_1) + 2\psi(0,q_1)\psi(1,q_1) - 1\psi(2,1+q_1) - 4\gamma\psi(0,q_2)/q_d$$
$$-2\psi(0,q_2)^2/q_d + 2\psi(1,1+q_2)/q_d + 2\gamma\psi(1,q_2) + 2\psi(0,q_2)\psi(1,q_2)$$
$$\left.-1\psi(2,1+q_2)\right) \tag{332}$$
$$q_d=\sqrt{12},\ q_1=(4-\sqrt{12})/2,\ q_2=(4+\sqrt{12})/2$$

$$\sum_{k=1}^{\infty}\frac{H(k)}{(k^2+5k+1)} = \frac{1}{2}\left(1/q_1^2/q_d - 2\gamma\psi(0,q_1)/q_d - 1\psi(0,q_1)^2/q_d + 1\psi(1,1+q_1)/q_d - 1/q_2^2/q_d\right.$$
$$\left.+2\gamma\psi(0,q_2)/q_d + 1\psi(0,q_2)^2/q_d - 1\psi(1,1+q_2)/q_d\right) \tag{333}$$
$$q_d=\sqrt{21},\ q_1=(5-\sqrt{21})/2,\ q_2=(5+\sqrt{21})/2$$

$$\sum_{k=1}^{\infty}\frac{H(k)}{(k^2+5k+1)^2} = \frac{1}{42}\left(-21168/q_1^2/q_d + 4\gamma\psi(0,q_1)/q_d + 2\psi(0,q_1)^2/q_d - 2\psi(1,1+q_1)/q_d + 966/q_1^3\right.$$
$$+2\gamma\psi(1,q_1) + 2\psi(0,q_1)\psi(1,q_1) - 1\psi(2,1+q_1) - 4\gamma\psi(0,q_2)/q_d$$
$$-2\psi(0,q_2)^2/q_d + 2\psi(1,1+q_2)/q_d + 2\gamma\psi(1,q_2) + 2\psi(0,q_2)\psi(1,q_2)$$
$$\left.-1\psi(2,1+q_2)\right) \tag{334}$$
$$q_d=\sqrt{21},\ q_1=(5-\sqrt{21})/2,\ q_2=(5+\sqrt{21})/2$$

$$\sum_{k=1}^{\infty}\frac{H(k)}{(k^2+5k+2)} = \frac{-1}{2}\left(-1/q_1^2/q_d + 2\gamma\psi(0,q_1)/q_d + 1\psi(0,q_1)^2/q_d - 1\psi(1,1+q_1)/q_d + 1/q_2^2/q_d\right.$$
$$\left.-2\gamma\psi(0,q_2)/q_d - 1\psi(0,q_2)^2/q_d + 1\psi(1,1+q_2)/q_d\right) \tag{335}$$
$$q_d=\sqrt{17},\ q_1=(5-\sqrt{17})/2,\ q_2=(5+\sqrt{17})/2$$

$$\sum_{k=1}^{\infty}\frac{H(k)}{(k^2+5k+2)^2} = \frac{1}{136}\left(-6647/q_1^2/q_d + 16\gamma\psi(0,q_1)/q_d + 8\psi(0,q_1)^2/q_d - 8\psi(1,1+q_1)/q_d + 714/q_1^3\right.$$
$$+8\gamma\psi(1,q_1) + 8\psi(0,q_1)\psi(1,q_1) - 4\psi(2,1+q_1) - 16\gamma\psi(0,q_2)/q_d$$
$$-8\psi(0,q_2)^2/q_d + 8\psi(1,1+q_2)/q_d + 8\gamma\psi(1,q_2) + 8\psi(0,q_2)\psi(1,q_2)$$
$$\left.-4\psi(2,1+q_2)\right) \tag{336}$$

$$q_d = \sqrt{17},\ q_1 = (5-\sqrt{17})/2,\ q_2 = (5+\sqrt{17})/2$$

$$\sum_{k=1}^{\infty} \frac{H(k)}{(k^2+5k+3)} = \frac{1}{2}\left(1/q_1^2/q_d - 2\gamma\psi(0,q_1)/q_d - 1\psi(0,q_1)^2/q_d + 1\psi(1,1+q_1)/q_d - 1/q_2^2/q_d\right.$$
$$\left.+2\gamma\psi(0,q_2)/q_d + 1\psi(0,q_2)^2/q_d - 1\psi(1,1+q_2)/q_d\right) \tag{337}$$

$$q_d = \sqrt{13},\ q_1 = (5-\sqrt{13})/2,\ q_2 = (5+\sqrt{13})/2$$

$$\sum_{k=1}^{\infty} \frac{H(k)}{(k^2+5k+3)^2} = \frac{-1}{702}\left(7436/q_1^2/q_d - 108\gamma\psi(0,q_1)/q_d - 54\psi(0,q_1)^2/q_d + 54\psi(1,1+q_1)/q_d - 1482/q_1^3\right.$$
$$-54\gamma\psi(1,q_1) - 54\psi(0,q_1)\psi(1,q_1) + 27\psi(2,1+q_1) + 108\gamma\psi(0,q_2)/q_d$$
$$+54\psi(0,q_2)^2/q_d - 54\psi(1,1+q_2)/q_d - 54\gamma\psi(1,q_2) - 54\psi(0,q_2)\psi(1,q_2)$$
$$\left.+27\psi(2,1+q_2)\right) \tag{338}$$

$$q_d = \sqrt{13},\ q_1 = (5-\sqrt{13})/2,\ q_2 = (5+\sqrt{13})/2$$

$$\sum_{k=1}^{\infty} \frac{H(k)}{(k^2+6k+1)} = \frac{1}{2}\left(1/q_1^2/q_d - 2\gamma\psi(0,q_1)/q_d - 1\psi(0,q_1)^2/q_d + 1\psi(1,1+q_1)/q_d - 1/q_2^2/q_d\right.$$
$$\left.+2\gamma\psi(0,q_2)/q_d + 1\psi(0,q_2)^2/q_d - 1\psi(1,1+q_2)/q_d\right) \tag{339}$$

$$q_d = \sqrt{32},\ q_1 = (6-\sqrt{32})/2,\ q_2 = (6+\sqrt{32})/2$$

$$\sum_{k=1}^{\infty} \frac{H(k)}{(k^2+6k+1)^2} = \frac{-1}{64}\left(71680/q_1^2/q_d - 4\gamma\psi(0,q_1)/q_d - 2\psi(0,q_1)^2/q_d + 2\psi(1,1+q_1)/q_d - 2176/q_1^3\right.$$
$$-2\gamma\psi(1,q_1) - 2\psi(0,q_1)\psi(1,q_1) + 1\psi(2,1+q_1) + 4\gamma\psi(0,q_2)/q_d$$
$$+2\psi(0,q_2)^2/q_d - 2\psi(1,1+q_2)/q_d - 2\gamma\psi(1,q_2) - 2\psi(0,q_2)\psi(1,q_2)$$
$$\left.+1\psi(2,1+q_2)\right) \tag{340}$$

$$q_d = \sqrt{32},\ q_1 = (6-\sqrt{32})/2,\ q_2 = (6+\sqrt{32})/2$$

$$\sum_{k=1}^{\infty} \frac{H(k)}{(k^2+7k+1)} = \frac{1}{2}\left(1/q_1^2/q_d - 2\gamma\psi(0,q_1)/q_d - 1\psi(0,q_1)^2/q_d + 1\psi(1,1+q_1)/q_d - 1/q_2^2/q_d\right.$$
$$\left.+2\gamma\psi(0,q_2)/q_d + 1\psi(0,q_2)^2/q_d - 1\psi(1,1+q_2)/q_d\right) \tag{341}$$

$$q_d = \sqrt{45},\ q_1 = (7-\sqrt{45})/2,\ q_2 = (7+\sqrt{45})/2$$

$$\sum_{k=1}^{\infty} \frac{H(k)}{(k^2+7k+1)^2} = \frac{-1}{90}\left(194400/q_1^2/q_d - 4\gamma\psi(0,q_1)/q_d - 2\psi(0,q_1)^2/q_d + 2\psi(1,1+q_1)/q_d - 4230/q_1^3\right.$$
$$-2\gamma\psi(1,q_1) - 2\psi(0,q_1)\psi(1,q_1) + 1\psi(2,1+q_1) + 4\gamma\psi(0,q_2)/q_d$$
$$+2\psi(0,q_2)^2/q_d - 2\psi(1,1+q_2)/q_d - 2\gamma\psi(1,q_2) - 2\psi(0,q_2)\psi(1,q_2)$$
$$\left.+1\psi(2,1+q_2)\right) \tag{342}$$

$$q_d = \sqrt{45},\ q_1 = (7-\sqrt{45})/2,\ q_2 = (7+\sqrt{45})/2$$

$$\sum_{k=1}^{\infty} \frac{H(k)}{(k^2+7k+2)} = \frac{1}{2}\left(1/q_1^2/q_d - 2\gamma\psi(0,q_1)/q_d - 1\psi(0,q_1)^2/q_d + 1\psi(1,1+q_1)/q_d - 1/q_2^2/q_d\right.$$
$$\left.+2\gamma\psi(0,q_2)/q_d + 1\psi(0,q_2)^2/q_d - 1\psi(1,1+q_2)/q_d\right) \tag{343}$$

$$q_d = \sqrt{41},\ q_1 = (7-\sqrt{41})/2,\ q_2 = (7+\sqrt{41})/2$$

$$\sum_{k=1}^{\infty}\frac{H(k)}{(k^2+7k+2)^2}=\frac{1}{328}\left(-79007/q_1^2/q_d+16\gamma\psi(0,q_1)/q_d+8\psi(0,q_1)^2/q_d-8\psi(1,1+q_1)/q_d+3690/q_1^3\right.$$
$$+8\gamma\psi(1,q_1)+8\psi(0,q_1)\psi(1,q_1)-4\psi(2,1+q_1)-16\gamma\psi(0,q_2)/q_d$$
$$-8\psi(0,q_2)^2/q_d+8\psi(1,1+q_2)/q_d+8\gamma\psi(1,q_2)+8\psi(0,q_2)\psi(1,q_2)$$
$$\left.-4\psi(2,1+q_2)\right) \tag{344}$$
$$q_d=\sqrt{41},\ q_1=(7-\sqrt{41})/2,\ q_2=(7+\sqrt{41})/2$$

$$\sum_{k=1}^{\infty}\frac{H(k)}{(k^2+7k+3)}=\frac{-1}{2}\left(-1/q_1^2/q_d+2\gamma\psi(0,q_1)/q_d+1\psi(0,q_1)^2/q_d-1\psi(1,1+q_1)/q_d+1/q_2^2/q_d\right.$$
$$\left.-2\gamma\psi(0,q_2)/q_d-1\psi(0,q_2)^2/q_d+1\psi(1,1+q_2)/q_d\right) \tag{345}$$
$$q_d=\sqrt{37},\ q_1=(7-\sqrt{37})/2,\ q_2=(7+\sqrt{37})/2$$

$$\sum_{k=1}^{\infty}\frac{H(k)}{(k^2+7k+3)^2}=\frac{-1}{1998}\left(125948/q_1^2/q_d-108\gamma\psi(0,q_1)/q_d-54\psi(0,q_1)^2/q_d+54\psi(1,1+q_1)/q_d\right.$$
$$-9546/q_1^3-54\gamma\psi(1,q_1)-54\psi(0,q_1)\psi(1,q_1)+27\psi(2,1+q_1)$$
$$+108\gamma\psi(0,q_2)/q_d+54\psi(0,q_2)^2/q_d-54\psi(1,1+q_2)/q_d-54\gamma\psi(1,q_2)$$
$$\left.-54\psi(0,q_2)\psi(1,q_2)+27\psi(2,1+q_2)\right) \tag{346}$$
$$q_d=\sqrt{37},\ q_1=(7-\sqrt{37})/2,\ q_2=(7+\sqrt{37})/2$$

$$\sum_{k=1}^{\infty}\frac{H(k)}{(k^2+7k+4)}=\frac{-1}{2}\left(-1/q_1^2/q_d+2\gamma\psi(0,q_1)/q_d+1\psi(0,q_1)^2/q_d-1\psi(1,1+q_1)/q_d+1/q_2^2/q_d\right.$$
$$\left.-2\gamma\psi(0,q_2)/q_d-1\psi(0,q_2)^2/q_d+1\psi(1,1+q_2)/q_d\right) \tag{347}$$
$$q_d=\sqrt{33},\ q_1=(7-\sqrt{33})/2,\ q_2=(7+\sqrt{33})/2$$

$$\sum_{k=1}^{\infty}\frac{H(k)}{(k^2+7k+4)^2}=\frac{1}{2112}\left(-49005/q_1^2/q_d+128\gamma\psi(0,q_1)/q_d+64\psi(0,q_1)^2/q_d-64\psi(1,1+q_1)/q_d+5412/q_1^3\right.$$
$$+64\gamma\psi(1,q_1)+64\psi(0,q_1)\psi(1,q_1)-32\psi(2,1+q_1)-128\gamma\psi(0,q_2)/q_d$$
$$-64\psi(0,q_2)^2/q_d+64\psi(1,1+q_2)/q_d+64\gamma\psi(1,q_2)+64\psi(0,q_2)\psi(1,q_2)$$
$$\left.-32\psi(2,1+q_2)\right) \tag{348}$$
$$q_d=\sqrt{33},\ q_1=(7-\sqrt{33})/2,\ q_2=(7+\sqrt{33})/2$$

$$\sum_{k=1}^{\infty}\frac{H(k)}{(k^2+7k+5)}=\frac{1}{2}\left(1/q_1^2/q_d-2\gamma\psi(0,q_1)/q_d-1\psi(0,q_1)^2/q_d+1\psi(1,1+q_1)/q_d-1/q_2^2/q_d\right.$$
$$\left.+2\gamma\psi(0,q_2)/q_d+1\psi(0,q_2)^2/q_d-1\psi(1,1+q_2)/q_d\right) \tag{349}$$
$$q_d=\sqrt{29},\ q_1=(7-\sqrt{29})/2,\ q_2=(7+\sqrt{29})/2$$

$$\sum_{k=1}^{\infty}\frac{H(k)}{(k^2+7k+5)^2}=\frac{-1}{7250}\left(74008/q_1^2/q_d-500\gamma\psi(0,q_1)/q_d-250\psi(0,q_1)^2/q_d+250\psi(1,1+q_1)/q_d\right.$$
$$-11310/q_1^3-250\gamma\psi(1,q_1)-250\psi(0,q_1)\psi(1,q_1)+125\psi(2,1+q_1)$$
$$+500\gamma\psi(0,q_2)/q_d+250\psi(0,q_2)^2/q_d-250\psi(1,1+q_2)/q_d-250\gamma\psi(1,q_2)$$
$$\left.-250\psi(0,q_2)\psi(1,q_2)+125\psi(2,1+q_2)\right)$$
$$q_d=\sqrt{29},\ q_1=(7-\sqrt{29})/2,\ q_2=(7+\sqrt{29})/2 \tag{350}$$